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Application of the Method of Decomposition into Exponential Series by the Spectral Parameter in Eigenvalue Problems

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Abstract. The Sturm-Liouville operator plays a central role in the theory of differential equations, mathematical physics, and applied mathematics. This operator arises in the Sturm-Liouville problem, which is an eigenvalue problem for a differential equation under consideration. The Sturm-Liouville operator generates a spectrum of eigenvalues and corresponding eigenfunctions. This is essential for solving partial differential equations through the separation of variables. The Sturm-Liouville theory is fundamental in understanding and solving linear differential equations with boundary conditions and serves as a bridge between pure and applied mathematics. The article explores the application of exponential series based on the spectral parameter to solve eigenvalue problems of Sturm-Liouville operators. A novel approach for decomposing the characteristic determinant into exponential series is proposed, demonstrating effectiveness in computing large eigenvalues. The asymptotic formulas for eigenvalues and eigenfunctions support the theoretical framework. Practical methods for achieving higher computational precision are also discussed. The work is based on an extension of earlier methods and offers new perspectives for numerical analysis in mathematical physics.

Keywords. Sturm-Liouville operator, spectral analysis, exponential series.

1 Introduction

In the paper [6], a method for decomposition into power series by the spectral parameter was proposed, which turned out to be effective for the numerical determination of the eigenvalues of the Sturm-Liouville operator. The problem of computing the eigenvalues of the

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Sturm-Liouville operator reduces to finding the zeros of the so-called characteristic determinant $\Delta(\lambda)$. The characteristic determinant of the Sturm-Liouville operator represents an entire function of the spectral parameter λ. Thus, the characteristic determinant $\Delta(\lambda)$ is decomposed into a power series by the spectral parameter λ with an infinite radius of convergence. In the paper [6], a simple method for finding the Taylor coefficients was provided. It turned out that the recurrence formulas for determining the Taylor coefficients give a simple and powerful method for numerically computing the eigenvalues. However, this approach is effective for calculating relatively small eigenvalues. For very large eigenvalues, the method proposed in [6] is not exactly ineffective, but for finding such eigenvalues, it is advisable to use exponential series by the spectral parameter. The exponential series we propose are effective for calculating sufficiently large eigenvalues in terms of magnitude. Exponential Series by the Spectral Parameter for the Sturm-Liouville Equation on a Segment.

The spectral properties of Sturm-Liouville operators have been analyzed in numerous studies. In particular, the works of Bondarenko [2], [3] investigate inverse problems for Sturm-Liouville operators, analyzing the sufficiency of information regarding the potential coefficients of the operator. Additionally, the studies by Law and Pivovarchik [4] on characteristic functions in quantum graphs are closely related to Sturm-Liouville theory. In this context, the works of Carlson and Pivovarchik [5] examine the spectral asymptotics of quantum graphs, investigating the fundamental conditions and regularities affecting the distribution of the operator's eigenvalues. Furthermore, a comprehensive review of quantum graphs and their applications is provided in the works of Berkolaiko, Carlson, Fulling, and Kuchment [7]. These studies contribute to the improvement of spectral analysis methods and enhance the understanding of their application in various operator systems. In this regard, the effectiveness of the proposed method is examined in comparison with the spectral characteristics of graphs, aiming to improve its capability in computing sufficiently large eigenvalues and to expand its applicability to other spectral problems.

2 Exponential Series by the Spectral Parameter for the Sturm-Liouville Equation on a Segment

Let us consider a second-order linear ordinary differential equation on a segment

$$-y'' + q(x)y = \lambda y, \quad 0 \le x \le b. \tag{1}$$

Such equations are called Sturm-Liouville equations. The coefficient $q(x)$ is often referred to as the potential. The conditions that the potential satisfies will be specified depending on the problem under study. The complex number λ plays the role of a spectral parameter. Often, instead of the parameter λ, it is convenient to use the parameter ρ, such that $\rho^2 = \lambda$.

Assume that λ is a complex number. Let $\varphi(x, \rho)$ denote the solution of the homogeneous equation (1), subjected to the Cauchy conditions at $x = b$.

$$\varphi(b,\rho) = 1, \quad \varphi'(b,\rho) = h, \tag{2}$$

where h is some complex constant.

According to the results of the monograph [1], the solution $\varphi(b,\rho)$ is the solution of the integral equation:

$$\varphi(x,\rho) = \cos\rho\,(x-b) \; + h\frac{\sin\rho\,(x-b)}{\rho} + \int\limits_x^b \frac{\sin\rho(x-b)}{\rho} q(t)\varphi(t,\rho)dt.$$

We define:

$$\varphi_0(x,\rho) = \cos\rho(x-b) + h\frac{\sin\rho(x-b)}{\rho}.$$

Let us assume:

$$\varphi_n(x,\rho) = \int\limits_x^b \frac{\sin\rho(x-b)}{\rho} q(t)\varphi_{n-1}(t,\rho)dt. \tag{3}$$

In the monograph [1], it was proven that the series $\sum_{k=0}^{\infty}\varphi_k(x,\rho)$ converges uniformly in λ for $|\lambda| \leq N$ and uniformly for $x \in [0;b]$. Here, N is an arbitrary positive number. Thus, the function $\varphi(x,\rho)$ is an entire function of the parameter ρ^2.

For further purposes, it is convenient to obtain the exponential representation for $\varphi_n(x,\rho)$. From relation (3), for a fixed natural n, we have the following equality:

$$\varphi_n(t_{n+1},\lambda) = \frac{1}{(\sqrt{\lambda})^n} \int\limits_b^{t_{n+1}} dt_n \cdots \int\limits_b^{t_3} dt_2 \int\limits_b^{t_2} dt_1 \prod_{i=1}^{n} q(t_i) \cdot$$
$$\cdot \prod_{i=1}^{n} \sin\sqrt{\lambda}(t_{i+1}-t_i) \left[\cos\sqrt{\lambda}(t_1-b) + \frac{h}{\lambda}\sin(t_1-b)\right], \quad n = 1,2,\ldots$$

where $t_{n+1} = x$.

Let I_n denote an n-dimensional unit parallelepiped, whose vertices are of the form $\varepsilon = (0,\ \varepsilon_2,\ \varepsilon_3,\ \ldots,\ \varepsilon_n)$. Here, the values of ε_i (for $i = \overline{1,n}$) can be either zero or one. Let a fixed vertex $\varepsilon \in I_n$, then let M_0 denote the number of equal neighboring coordinate pairs. In other words,

$$M_0 = \text{card}\left\{\exists i \in [1,n] \cap \mathbb{N} : \varepsilon_i = \varepsilon_{i+1}\right\}.$$

Using the identity

$$\prod_{i=1}^{n} \sin\sqrt{\lambda}\,(t_{i+1}-t_i) =$$
$$= \begin{cases} \dfrac{4}{4^k} \displaystyle\sum_{(0\varepsilon_1\varepsilon_2\ldots\varepsilon_{2k-1})\epsilon I_{2k-1}} (-1)^{k-1+\sum_{S=1}^{2k-1}\varepsilon_s} \sin\sqrt{\lambda}\sum_{i=1}^{2k-1}(-1)^{\varepsilon_i}(t_{i+1}-t_i), & \neg n = 2k-1, \\ \dfrac{2}{4^k} \displaystyle\sum_{(0\varepsilon_1\varepsilon_2\ldots\varepsilon I_{2k})\epsilon I_{2k}} (-1)^{k+\sum_{S=1}^{2k}\varepsilon_s} \cos\sqrt{\lambda}\sum_{i=1}^{2k}(-1)^{\varepsilon_i}(t_{i+1}-t_i), & \neg n = 2k. \end{cases} \tag{4}$$

For $n = 2k-1$ and $n = 2k$, from equation (4), we derive the required exponential representations. To do this, we introduce the quantities for $n = 2k-1$:

$$\begin{aligned}
\min\tau_{1(2k-1)} &= \left((-1)^{\varepsilon_{2k-1}} + 2\left(k-1+A\left[\frac{M_0}{2}\right]-M_0\right)\right)x - \left(2\left(k-A\left[\frac{M_0}{2}\right]\right)-1\right)b,\\
\max\tau_{1(2k-1)} &= \left((-1)^{\varepsilon_{2k-1}} - 2\left(k-1-A\left[\frac{M_0}{2}\right]\right)\right)x + \left(2\left(k+A\left[\frac{M_0}{2}\right]-M_0\right)-3\right)b,\\
\min\tau_{2(2k-1)} &= \left(-(-1)^{\varepsilon_{2k-1}} + 2\left(k+A\left[\frac{M_0}{2}\right]-M_0\right)\right)x - \left(2\left(k-A\left[\frac{M_0}{2}\right]\right)-1\right)b,\\
\max\tau_{2(2k-1)} &= \left(-(-1)^{\varepsilon_{2k-1}} - 2\left(k-1-A\left[\frac{M_0}{2}\right]\right)\right)x + \left(2\left(k+A\left[\frac{M_0}{2}\right]-M_0\right)+1\right)b,
\end{aligned}$$

for $n = 2k$:

$$\begin{aligned}
\min\tau_{1(2k)} &= \left((-1)^{\varepsilon_{2k-1}} + 2\left(k-A\left[\frac{M_0}{2}\right]\right)\right)x - \left(2\left(k-A\left[\frac{M_0}{2}\right]-M_0\right)-1\right)b,\\
\max\tau_{1(2k)} &= \left((-1)^{\varepsilon_{2k-1}} + 2\left(k-1-A\left[\frac{M_0}{2}\right]-M_0\right)\right)x + \left(2\left(k+A\left[\frac{M_0}{2}\right]\right)-1\right)b,\\
\min\tau_{2(2k)} &= \left(2-(-1)^{\varepsilon_{2k-1}} + 2\left(k-A\left[\frac{M_0}{2}\right]\right)\right)x - \left(2\left(k+A\left[\frac{M_0}{2}\right]-M_0\right)-1\right)b,\\
\max\tau_{2(2k)} &= \left(-(-1)^{\varepsilon_{2k-1}} - 2\left(k-1+A\left[\frac{M_0}{2}\right]-M_0\right)\right)x - \left(2\left(k-A\left[\frac{M_0}{2}\right]\right)+1\right)b.
\end{aligned}$$

Lemma 1. *Let k be a fixed natural number. Then there exist continuous functions with respect to τ, $A^{M_0}_{1(2k-1)}(x,\tau_{1(2k-1)},b), A^{M_0}_{2(2k-1)}\left(x,\tau_{2(2k-1)},b\right), B_{2k-1}(x)$ such that the following* *exponential representation holds:*

$$\varphi_{2k-1}(x,\rho)=\sum_{\epsilon_{2k-1}=0}^{1}\sum_{M_0=0}^{2k-1}C_{2k-2}^{M_0}\Bigg[\frac{1}{\rho^{2k-1}}\int_{\min\tau_{1(2k-1)}}^{\max\tau_{1(2k-1)}}A_{1(2k-1)}(x,\tau_{1(2k-1)},b)\sin\rho\tau_{1(2k-1)}d\tau_{1(2k-1)}$$
$$-\frac{1}{\rho^{2k-1}}\int_{\min\tau_{2(2k-1)}}^{\max\tau_{2(2k-1)}}A_{2(2k-1)}^{M_0}(x,\tau_{2(2k-1)},b)\sin\rho\tau_{2(2k-1)}d\tau_{2(2k-1)}$$
$$+\frac{h}{\rho^{2k}}\int_{\min\tau_{2(2k-1)}}^{\max\tau_{2(2k-1)}}A_{2(2k-1)}^{M_0}(x,\tau_{2(2k-1)},b)\cos\rho\tau_{2(2k-1)}d\tau_{2(2k-1)}$$
$$-\frac{h}{\rho^{2k}}\int_{\min\tau_{1(2k-1)}}^{\max\tau_{1(2k-1)}}A_{1(2k-1)}^{M_0}(x,\tau_{1(2k-1)},b)\cos\rho\tau_{1(2k-1)}d\tau_{1(2k-1)}\Bigg]$$
$$+\frac{1}{\rho^{2k-1}}B_{2k-1}(x)\sin\rho(x-b)-\frac{1}{\rho^{2k-1}}\int_{b-x}^{x-b}A_{2(2k-1)}^{2k-1}(x,\tau_{2(2k-1)},b)\sin\rho\tau_{2(2k-1)}d\tau_{2(2k-1)}$$
$$-\frac{h}{\rho^{2k}}B_{2k-1}(x)\cos\rho(x-b)+\frac{h}{\rho^{2k-1}}\int_{b-x}^{x-b}A_{2(2k-1)}^{2k-1}(x,\tau_{2(2k-1)},b)\cos\rho\tau_{2(2k-1)}d\tau_{2(2k-1)}.$$

Lemma 2. *Let k be a fixed natural number. Then there exist continuous functions with respect to τ, $A_{1(2k)}^{M_0}(x,\tau_{1(2k)},b), A_{2(2k)}^{M_0}\left(x,\tau_{2(2k)},b\right), B_{2k}(x)$, such that the following exponential representation holds:*

$$\varphi_{2k}(x,\rho)=\sum_{\varepsilon_{2k-1}=0}^{1}\sum_{M_0=0}^{2k-3}C_{2k-1}^{M_0}\Bigg[\frac{1}{\rho^{2k}}\int_{min\tau_{1(2k)}}^{max\tau_{1(2k)}}A_{1(2k)}(x,\tau_{1(2k)},b)\cos\rho\tau_{1(2k)}d\tau_{1(2k)}$$
$$+\frac{1}{\rho^{2k}}\int_{min\tau_{2(2k)}}^{max\tau_{2(2k)}}A_{2(2k)}^{M_0}(x,\tau_{2(2k)},b)\cos\rho\tau_{2(2k)}d\tau_{2(2k)}$$
$$+\frac{h}{\rho^{2k+1}}\int_{min\tau_{2(2k)}}^{max\tau_{2(2k)}}A_{2(2k)}^{M_0}(x,\tau_{2(2k)},b)\sin\rho\tau_{2(2k)}d\tau_{2(2k)}$$
$$-\frac{h}{\rho^{2k+1}}\int_{min\tau_{1(2k)}}^{max\tau_{1(2k)}}A_{1(2k)}^{M_0}(x,\tau_{1(2k)},b)\sin\rho\tau_{1(2k)}d\tau_{1(2k)}\Bigg]$$
$$+\frac{1}{\rho^{2k}}B_{2k}(x)\cos\rho(x-b)-\frac{1}{\rho^{2k}}\int_{b-x}^{x-b}A_{2(2k-1)}^{2k}(x,\tau_{2(2k)},b)\cos\rho\tau_{2(2k)}d\tau_{2(2k)}$$
$$+\frac{h}{\rho^{2k+1}}B_{2k}(x)\sin\rho(x-b)+\frac{h}{\rho^{2k+1}}\int_{b-x}^{x-b}A_{2(2k-1)}^{2k}(x,\tau_{2(2k)},b)\sin\rho\tau_{2(2k)}d\tau_{2(2k)}.$$

Using Lemmas 1 and 2, the required solution of Problem (1)–(2) can be written as an exponential series in terms of the spectral parameter.

Theorem 3. *Let $q \in C[a,b]$. Then, for any complex λ, the solution to the problem (1)–(2) exists and is unique, and moreover, the following representation is valid:*

$$\begin{aligned}
\varphi(x,\rho) = &\cos\rho(x-b) + \frac{1}{\rho}(h + B_1(x))\sin(x-b) \\
&- \frac{1}{\rho}\int_{b-x}^{x-b} A_{23}^1(x,\tau_{23},b)\sin\rho\tau_{23}d\tau_{23} \\
&- \sum_{k=1}^{\infty}\Bigg\{\frac{1}{\rho^{2k}}(B_{2k}(x) - hB_{2k+1}(x))\cos\rho(x-b) \\
&+ \frac{1}{\rho^{2k}}\sum_{\varepsilon_{2k-1}=0}^{1}\Bigg(\sum_{M_0=0}^{2k-3} C_{2k-1}^{M_0}\int_{min\tau_{1(2k-1)}}^{max\tau_{1(2k-1)}} A_{2(2k+1)}(x,\tau_{2(2k+1)},b)\cos\rho\tau_{1(2k-1)}d\tau_{1(2k+1)} \\
&+ \sum_{M_0=0}^{2k-2} C_{2k-1}^{M_0}\int_{min\tau_{2(2k)}}^{max\tau_{2(2k)}} A_{2(2k)}^{M_0}(x,\tau_{2(2k)},b)\cos\rho\tau_{2(2k)}d\tau_{2(2k)}\Bigg) \\
&+ h\int_{b-x}^{x-b} A_{2(2k-1)}^{2k+1}(x,\tau_{2(2k-1)},b)\cos\rho\tau_{2(2k-1)}d\tau_{2(2k-1)} \\
&+ \int_{b-x}^{x-b} A_{2(2k-1)}^{2k}(x,\tau_{2(2k)},b)\cos\rho\tau_{2(2k)}d\tau_{2(2k)} \\
&+ \frac{1}{\rho^{2k+1}}(B_{2k+1}(x) + hB_{2k}(x))\sin\rho(x-b) \\
&+ \frac{1}{\rho^{2k+1}}\sum_{\varepsilon_{2k-1}=0}^{1}\Bigg(\sum_{M_0=0}^{2k-3} C_{2k-2}^{M_0}\int_{min\tau_{1(2k-1)}}^{max\tau_{1(2k-1)}} A_{1(2k+1)}(x,\tau_{1(2k+1)},b)\sin\rho\tau_{1(2k+1)}d\tau_{1(2k+1)} \\
&- \sum_{M_0=0}^{2k-3} C_{2k-2}^{M_0}\int_{min\tau_{2(2k+1)}}^{max\tau_{2(2k+1)}} A_{2(2k+1)}^{M_0}(x,\tau_{2(2k+1)},b)\sin\rho\tau_{2(2k+1)}d\tau_{2(2k+1)} \\
&+ \sum_{M_0=0}^{2k-2} C_{2k-1}^{M_0}\int_{min\tau_{2(2k)}}^{max\tau_{2(2k)}} A_{2(2k)}^{M_0}(x,\tau_{2(2k)},b)\sin\rho\tau_{2(2k)}d\tau_{2(2k)} \\
&+ h\sum_{M_0=0}^{2k-2} C_{2k-1}^{M_0}\int_{min\tau_{1(2k)}}^{max\tau_{1(2k)}} A_{1(2k)}^{M_0}(x,\tau_{1(2k)},b)\sin\rho\tau_{1(2k)}d\tau_{1(2k)}\Bigg) \\
&- \int_{b-x}^{x-b} A_{2(2k+1)}^{2k-1}(x,\tau_{2(2k+1)},b)\sin\rho\tau_{2(2k+1)}d\tau_{2(2k+1)} \\
&+ h\int_{b-x}^{x-b} A_{2(2k-1)}^{2k}(x,\tau_{2(2k)},b)\sin\rho\tau_{2(2k)}d\tau_{2(2k)}\Bigg\}.
\end{aligned} \tag{5}$$

3 Asymptotic formulas for eigenvalues and eigenfunctions

Now we derive the asymptotic formulas for eigenvalues and eigenfunctions. From these formulas, in particular, it follows that an infinite set of eigenvalues exists.

We still assume at the beginning that $h \neq \infty$ and $H \neq \infty$. For any λ, the function $\varphi(x,\lambda)$ obviously satisfies the first boundary condition (1)–(2). Therefore, we will determine the eigenvalues if we substitute the function $\varphi(x,\lambda)$ into the second boundary condition.

According to Lemma 1-2 from [1], the eigenvalues are real, i.e. $Im\,\rho = 0$. Therefore, we estimate the series (5) as follows:

$$\varphi(x,\rho) = cos\rho\,(x-b) + \frac{1}{\rho}\,(h + B_1(x))\,sin\rho\,(x-b) - \frac{1}{\rho}\int_{b-x}^{x-b} A_{23}^1(x,\tau_{23},b)\,sin\rho\tau d\tau + \xi_1. \quad (5')$$

Next, differentiating equation (5') with respect to x and using the estimate (5'), it is easy to obtain the following estimate:

$$\begin{aligned}\varphi'_x(x,\rho) =& \rho\sin\rho(x-b) + (h + B_1(x))\cos\rho(x-b) - A_{23}^1(x,\tau_{23},b)\frac{\sin\rho\tau}{\rho}\\ &+ \frac{1}{\rho}\int_b^x A_{23}^1(x,\tau_{23},b)_x\frac{\sin\rho\tau}{\rho}d\tau + \xi_1.\end{aligned} \quad (6)$$

Now, substituting the values of the functions $\varphi(x,\rho)$ and $\varphi'_x(x,\rho)$ from estimates (5') and (6) into the second boundary condition (2), we obtain the following equation for determining the eigenvalues:

$$\begin{gathered}\cos\rho b - (h + B_1(x))\,\frac{\sin\rho\tau}{\rho} + \int_{-b}^{b} A_{23}^1(0,\tau_{23},b)\frac{\sin\rho\tau}{\rho}d\tau + \xi_1 = 0\\ \rho\to\infty:\ \cos\rho b = 0,\ \rho_0 = \frac{\pi}{2b}(2m+1),\ m\in\mathbb{Z}.\end{gathered} \quad (7)$$

We look for the root in the form $\rho = \frac{\pi}{2b}(2m+1) + \delta(m),\ m\in\mathbb{Z}$. Then from equation (7), we have the following relationship:

$$\begin{aligned}&\cos\left(\frac{\pi}{2}(2m+1) + b\delta\right) - (h + B_1(0))\left(\frac{\pi}{2b}(2m+1) + \delta\right)^{-1}\sin\left(\frac{\pi}{2}(2m+1) + b\delta\right)\\ &+ \int_{-b}^{b} A_{23}^1(0,\tau_{23},b)\left(\frac{\pi}{2b}(2m+1) + \delta\right)^{-1}\sin\left(\frac{\pi}{2}(2m+1) + b\delta\right)d\tau + \xi_1 = 0\end{aligned}$$

or

$$\begin{aligned}&(-1)^{m+1}\sin b\delta + (-1)^{m+1}(h + B_1(0))\left(\frac{\pi}{2b}(2m+1) + \delta\right)^{-1}\cos b\delta\\ &+ (-1)^m\int_{-b}^{b} A_{23}^1(0,\tau_{23},b)\left(\frac{\pi}{2b}(2m+1) + \delta\right)^{-1}\cos b\delta d\tau + \xi_1 = 0\end{aligned} \quad (8)$$

From this, as $m \to \infty$, we get the limiting relation

$$\lim_{m\to\infty} \sin b\delta(m) = 0,$$

which is equivalent to the following equality:

$$\lim_{m\to\infty} \delta(m) = 0 \tag{9}$$

From the relation (8), taking into account the limiting equality (9), we have

$$(-1)^{m+1}\sin b\delta + (-1)^{m+1}(h + B_1(0))\left(\frac{\pi}{2b}(2m+1)+\delta\right)^{-1}\cos b\delta$$
$$+ (-1)^m \int_{-b}^{b} A_{23}^1(0,\tau_{23},b)\left(\frac{\pi}{2b}(2m+1)+\delta\right)^{-1}\cos b\delta d\tau + \xi_1 = 0$$

Thus, we find an approximate value of $\delta(m)$. This process of refining the root computation can continue to the desired level of accuracy. Therefore, the obtained approximate value can be used as a solution with the required accuracy. Continuing the process of refining the root computation, we can achieve even greater accuracy.

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Қангужин Б.Е., Хужахметов Ж.Ж. СПЕКТРАЛЬДЫҚ ПАРАМЕТР БОЙЫНША ЭКСПОНЕНЦИАЛДЫ ҚАТАРЛАРҒА ЖІКТЕУ ӘДІСІН МЕНШІКТІ МӘНДЕР ЕСЕПТЕРІНДЕ ҚОЛДАНУ

Штурм-Лиувилл операторы дифференциалдық теңдеулер теориясында, математикалық физикада және қолданбалы математикада орталық рөл атқарады. Бұл оператор Штурм-Лиувилл есебінде туындайды, ол қарастырылып отырған дифференциалдық теңдеу үшін меншікті мән есебі болып табылады. Штурм-Лиувилль операторы меншікті мәндер мен сәйкес меншікті функциялардың спектрін жасайды. Бұл айнымалыларды бөлу арқылы дербес дифференциалдық теңдеулерді шешу үшін өте қажет. Штурм-Лиувилл теориясы шекаралық шарттары бар сызықтық дифференциалдық теңдеулерді түсіну және шешу үшін негіз болып табылады және таза математика мен қолданбалы математика арасындағы көпір қызметін атқарады.

Бұл мақалада Штурм-Лиувилль операторларының меншікті мәндер есебін шешу үшін спектрлік параметр бойынша экспоненциалды қатарларды қолдану қарастырылады. Сипаттамалық анықтауышы экспоненциалды қатарларға жіктеудің жаңа тәсілі ұсынылып, үлкен меншікті мәндерді есептеуде тиімділігі көрсетіледі. Меншікті мәндер мен меншікті функциялар үшін асимптотикалық формулалар теориялық негізді растайды. Сондай-ақ, есептеу дәлдігін арттырудың практикалық әдістері талқыланады. Жұмыс бұрынғы әдістердің кеңейтілуіне негізделген және математикалық физикадағы сандық талдау үшін жаңа көзқарастар ұсынады.

Түйін сөздер: Штурм-Лиувилль операторы, спектрлік талдау, экспоненциалды қатарлар.

Кангужин Б.Е., Хужахметов Ж.Ж. ПРИМЕНЕНИЕ МЕТОДА РАЗЛОЖЕНИЯ В ЭКСПОНЕНЦИАЛЬНЫЕ РЯДЫ ПО СПЕКТРАЛЬНОМУ ПАРАМЕТРУ В ЗАДАЧАХ НА СОБСТВЕННЫЕ ЗНАЧЕНИЯ

Оператор Штурма-Лиувилля играет центральную роль в теории дифференциальных уравнений, математической физике и прикладной математике. Этот оператор возникает в задаче Штурма-Лиувилля, которая является задачей на собственные значения для рассматриваемого дифференциального уравнения. Оператор Штурма-Лиувилля генерирует спектр собственных значений и соответствующих собственных функций. Это необходимо для решения уравнений в частных производных путем разделения переменных. Теория Штурма-Лиувилля является основополагающей для понимания и решения линейных дифференциальных уравнений с граничными условиями и служит мостом между чистой и прикладной математикой.

В данной статье исследуется применение экспоненциальных рядов по спектральному параметру для решения задач на собственные значения операторов Штурма-Лиувилля.

Предлагается новый подход к разложению характеристического определителя в экспоненциальные ряды, демонстрирующий эффективность при вычислении больших собственных значений. Асимптотические формулы для собственных значений и собственных функций подтверждают теоретическую основу. Также обсуждаются практические методы достижения более высокой вычислительной точности. Работа основана на расширении более ранних методов и предлагает новые перспективы для численного анализа в математической физике.

Ключевые слова: оператор Штурма-Лиувилля, спектральный анализ, экспоненциальные ряды.