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Transformation of degenerate indirect control systems in the vicinity of a program manifold

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Abstract. We consider one of the classes of implicit differential systems, systems of ordinary differential equations that are not resolved with respect to the highest derivative. Such equations are often found in everyday life in mechanics, physics, economics, biology, etc. The problems of constructing automatic control systems according to a given smooth program manifold are also come down to such equations.This is the case when the dimension of the systems of equations under construction is greater than the dimension of the program manifold. Then systems of algebraic equations with a rectangular matrix arise. We consider a system with a square matrix, the discriminant of which is zero. The general problem of constructing systems of differential equations for a given manifold is considered. A necessary and sufficient condition is drawn up that the manifold is integral to the system of equations. The Yerugin function is linear with respect to the manifold.Then an indirect control system is built, taking into account that a given manifold is integral to it under certain conditions. In general, the Jacobi matrix is rectangular. The case is investigated when the matrix is quadratic and has zero roots. The manifold is assumed to be linear with respect to the desired variable. A degenerate indirect control system is obtained, unresolved with respect to the highest derivative. Equivalence to a certain system is established, the matrices of which are constant and have a special structure. Lyapunov transformation matrices are found. It is shown that the considered control systems can be reduced to a central canonical form. A brief overview is provided.

Keywords. program manifold, degenerate systems, equivalence of systems, indirect control automatic systems, Lyapunov transformation, canonical forms.

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1 Introduction

We consider an implicit unsolved system with respect to the derivative

$$H((t,x(t))\dot{x}(t) = F(t,x), \quad H \in R^{s\times n},\ x \in R^n,\ F \in R^s,\ t \in I = (-\alpha,\ \beta)\,. \tag{1}$$

Here α, β are finite or infinite numbers. Linear systems of this kind unsolved with respect to high derivative or algebraic differential systems have wide application in everyday practice. These systems have substantial applications in the dynamic of a space vehicle, economic control, robotics, theory of electric chains ets. Here is a brief review of some important properties of these systems. The existence and uniqueness of solutions of degenerate linear systems, and reducibility of systems with variable matrices to the systems with constant matrices were studied by A.M. Samoilenko and V.P. Yacovets [1], V.P. Yacovets [2]. In these works degenerate systems were reduced to different canonical forms, and solution algorithms were constructed for linear systems. In work [3], S.A. Mazanic examined the equivalence problem considering systems to systems with constant and piecewise constant coefficients. Central canonical form and stability of degenerate control systems were considered in [4].

The problem of constructing systems based on a given program manifold is also a subclass for systems of type (1). We investigate the establishment problem of equivalence and reducibility degenerate indirect automatic control systems.

Consider the problem of constructing, for a given smooth program manifold $\Omega(t)$, the following system of differential equations

$$\dot{x} = f\,(t, x)\,, \tag{2}$$

where f, x are n-dimensional vectors, $f \in R^n$ is continuous in all variables and the existence conditions of the solution $x(t) = 0$ are satisfied; and the program manifold $\Omega(t)$ is defined by the following equations

$$\Omega\,(t) \equiv \omega(t, x) = 0, \tag{3}$$

where an s-dimensional vector ω $(s \leq n)$ is continuous in the single-connected closed domain including the manifold $\Omega(t)$, together with its partial derivatives.

Definition 1. The set $\Omega(t)$ is called an integral manifold of the equation (2), if the condition $\omega(t_0, x_0) \in \Omega(t_0)$ implies that $\omega(t, x) \in \Omega(t)$ for all $t > t_0$.

Note that the term "program manifold", used in this paper, is equivalent to the notion of "integral manifold".

Composing a necessary and sufficient condition that the program manifold $\Omega(t)$ is integral for the system (2) we obtain

$$\dot{\omega} = \frac{\partial \omega}{\partial t} + H\dot{x} = F(t, x, \omega), \tag{4}$$

where $F(t,x,0) \equiv 0$ is some Erugin vector function [5, 6], $H = \dfrac{\partial \omega}{\partial x}$ is the Jacobian matrix and its rank is equal to $rankH = s$ at all points of $\Omega(t)$.

Solving equations with respect to $\dot{x}$ we find

$$H((t,x(t))\dot{x}(t) = F(t,x,\omega) - \frac{\partial \omega}{\partial t}, t \in I = (-\alpha,\ \beta)\,. \tag{5}$$

$$H \in R^{s\times n}, x \in R^n, \omega \in R^s, F \in R^s.$$

At $s < n$ many authors have been studying the construction of equations systems on a given program manifold possessing by the stability properties, optimality, and establishing quality estimates of transition process index in the vicinity of the manifold. A detailed review of these investigations is adduced in [7–9]. We consider the problem of finding transformation matrices for the system (5) allowing us to reduce them to an equivalent system.

2 Transformation of degenerate indirect control systems in the vicinity of program manifold.

Together with Equation (2), we consider the indirect control system with feedback of the following structure [10]:

$$\begin{array}{l} \dot{x} = f\,(t,x) - B_1\varphi(\sigma), \quad t \in I = (-\alpha,\ \beta)\,, \\ \dot{\xi} = \varphi\,(\sigma)\,, \quad \sigma = P^T\omega - Q\xi, \end{array} \tag{6}$$

where $x \in R^n$ is a state vector of the object, $f \in R^n$ is a vector-function, satisfying to conditions of existence of a solution $x(t) = 0$, and $B_1 \in R^{n\times r}$, $P \in R^{s\times r}$ are constant matrices, $Q \in R^{r\times r}$ is a constant matrix of rigid feedback, $\varphi(\sigma)$ is a function differentiable with respect to σ, satisfies the following conditions

$$\varphi(0) = 0 \wedge 0 < \sigma^T\varphi(\sigma) < \sigma^T K\sigma \quad \forall\sigma \neq 0. \tag{7}$$

Here $K = K^T > 0$, $K \in R^{r\times r}$.

For the manifold $\Omega(t)$ to be integral also for the system (6)–(7) on the manifold $\omega = 0$ it is necessary to have the condition $\xi = 0$. This condition is satisfied if and only if $Q \neq 0$.

Taking into account that $\Omega(t)$ is an integral for the system (6)–(7) differentiating program manifold $\Omega(t)$ (3) in time t by virtue of the system (6), we obtain

$$\begin{array}{l} H((t,x(t))\dot{x}(t) = F(t,x,\omega) - q_1(t) - B\xi, \\ \dot{\xi} = \varphi\,(\sigma)\,, \quad \sigma = P^T\omega - Q\xi, \quad t \in I = (\alpha,\ \beta)\,, \end{array} \tag{8}$$

where $H = \dfrac{\partial \omega}{\partial x}$, $q_1 = \dfrac{\partial \omega}{\partial t}$, $B = HB_1$, $H \in R^{s\times n}$, $x \in R^n$, $\omega \in R^s$, $F \in R^s$, nonlinearity $\varphi(\sigma)$ satisfies also to generalized conditions (7).

We consider the case where $s = n$ and the matrix H has k null roots. Choosing the manifold in the following form

$$\omega = A_1(t)x + g(t) = 0, \tag{9}$$

where $A_1(t) \in R^{s\times s}$ is a given continuous matrix, $g(t)$ is a continuous vector function, we present the Erugin function in the form of

$$F(t, x, \omega) = -A_2(t)x. \tag{10}$$

Here $-A_2$ is a Hurwitz matrix, $A_2 \in R^{s\times s}$.

Thus we obtain the following system:

$$\begin{aligned} &H(t)\dot{x}(t) = -A(t)x - q(t) - B\xi, \\ &\dot{\xi} = \varphi(\sigma), \quad \sigma = \Pi^T x - P^T g(t) - Q\xi, \quad t \in I = (\alpha,\ \beta), \end{aligned} \tag{11}$$

where $H(t) = A_1(t)$, $A(t) = -A_2(t)A_1(t) - \dfrac{\partial A_1(t)}{\partial t}$, $q(t) = \dfrac{\partial g(t)}{\partial t} + A_1(t)g(t)$, $\Pi^T = P^T A_1(t)$.

In (11) we select the linear part relatively to x:

$$H(t)\dot{x}(t) = -A(t)x. \tag{12}$$

Definition 2. An absolutely continuous function $x(t)$ is called a *solution of System* (12) if it makes the identity of this system almost everywhere in the interval $t \in I$.

Definition 3. An absolutely continuous matrix $X \in R^{s\times r}$ is called *a fundamental matrix* of System (12) if for all constant vectors $c \in R^r$ a function $x(t) = X(t)c$ is a solution of system (12) and for any solution $x(t)$ of System (12) there exists a unique constant vector c such that $x(t) = X(t)c$.

We consider a system of a similar type together with the system (12):

$$D(t)\dot{y} + G(t)y = 0, t \in I, \tag{13}$$

where D and G are absolutely continuous $(s\times s)$-dimensional matrices bounded on the interval I, for all $t \in I$ the determinant of the matrix D is equal to zero and the automatic control system has the following form

$$\begin{aligned} &D(t)\dot{y} = G(t)y - \overline{q}(t) - \overline{B}\xi, \\ &\dot{\xi} = \varphi(\sigma), \quad \sigma = \overline{G}(t)y - \overline{G}g(t) - \overline{Q}\xi, \quad t \in I = (\alpha,\ \beta). \end{aligned} \tag{14}$$

Here $\overline{B} \in R^{n\times\nu}$, $\overline{G} \in R^{s\times\nu}$, $\overline{Q} \in R^{\nu\times\nu}$ are constant matrices, nonlinearity $\varphi(\sigma)$ satisfies conditions of the type (7).

Definition 4. Systems (12) and (13) are called *asymptotical equivalent* if there exists a Lyapunov matrix L such that for any solution y of System (13) and a function $x = Ly$ is a solution of System (12), and for any solution x of System (12) the function $y = L^{-1}x$ is a solution of System (13).

Theorem 5. *Systems (12) and (13) are* equivalent *if and only if there exists a Lyapunov matrix L such that for the fundamental matrix Y of a solution to System (13) one can find a fundamental matrix X of a solution to System (12) for which the presentation $X = LY$ is valid.*

Theorem 6. *Let $H(t)$ and $A(t)$ are absolute continuous matrices bounded together with their first derivatives in the interval I, $rank H(t) = k$ for all $t \in I$ and for all $k, 1 < k < s$ and there is a sub-matrix $H_0(t) \in R^{k\times k}$ of the matrix $H(t)$ satisfying the following conditions*

$$\inf[\det(H_0(t)] > 0 \quad \forall t \in I, \ \inf[\partial^r/\partial\lambda^r \det(H(t)\lambda + A(t)] > 0 \quad \forall t \in I. \tag{15}$$

Then for all $t \in I$ there exist non-singular matrices T and S such that multiplied by T the left-hand side and replaced by $x = Sz$ System (12) is reduced to the equivalent system (13) and the matrices $D(t)$ and $G(t)$ are of the form:

$$D(t) = \left\| \begin{array}{cc} O_1 & O_2 \\ O_3 & E_0 \end{array} \right\|, \quad G(t) = \left\| \begin{array}{cc} E_1 & O_2 \\ O_3 & G_0(t) \end{array} \right\|, \tag{16}$$

where O_1, O_2, and O_3 are $(k \times k)$, $(k \times r)$, and $(r \times k)$-dimensional null matrices, correspondingly, E_1 and E_0 are $(k \times k)$ and $(r \times r)$-dimensional unique matrices, $G_0(t)$ is a local summable and bounded $(r \times r)$-dimensional matrix.

Proof. Let the submatrix $H_0(t)$ be in the lower right angle of the matrix $H(t)$. We represent the matrix $H(t)$ in the block form.

$$H(t) = \left\| \begin{array}{cc} H_1(t) & H_2(t) \\ H_3(t) & H_0(t) \end{array} \right\|, \tag{17}$$

where $H_1(t)((k \times k)$, $H_2(t)((k \times r)$, and $H_3(t)((r \times k)$ are matrices. Then there exist absolute continuous in the interval I matrices $C_1(t)((k \times r)$ and $C_3(t)((r \times k)$ which are bounded together with their first derivatives

$$C_1(t) = H_2(t)H_0^{-1}(t), \quad C_3(t) = H_0^{-1}(t)H_3(t).$$

Therefore, the matrix $H(t)$ can be represented in the following form:

$$H(t) = \left\| \begin{array}{ll} C_1(t)H_0(t)C_3(t) & C_1(t)H_0(t) \\ H_0(t)C_3(t) & H_0(t) \end{array} \right\|. \tag{18}$$

Choosing matrices $T(t)$ and $S(t)$ in the form of

$$T(t) = \left\| \begin{array}{cc} E_1 & -C_1(t) \\ O_1 & H_0^{-1}(t) \end{array} \right\|, \quad S(t) = \left\| \begin{array}{cc} E_1 & O_2 \\ -C_3(t) & E_0 \end{array} \right\|, \tag{19}$$

and multiplying by T the left hand said of System (11) and replacing by $x = S(t)z$ we obtain

$$\begin{array}{l} D(t)\dot{z} = -F(t)z - q(t) - D(t)B\xi, \\ \dot{\xi} = \varphi(\sigma), \quad \sigma = D^T(t)A_1(t)S(t)z - D^T g(t) - Q\xi, \quad t \in I = (\alpha, \ \beta). \end{array} \tag{19}$$

Here D is the same (16) and

$$F(t) = T(t)H(t)\dot{S}(t) + BS(t). \tag{20}$$

According to the definition of the matrix $C_3(t)$ we conclude that $S(t)$ is a Lyapunov matrix and System (19) is asymptotically equivalent to System (12). Consequently, System (19) is equivalent to System (11).

Now we represent the matrix $F(t)$ in the bloc form.

$$F(t) = \left\| \begin{array}{cc} F_1(t) & F_2(t) \\ F_3(t) & F_0(t) \end{array} \right\|, \tag{21}$$

where $F_1(t)((k \times k), F_2(t)((k \times r), F_3(t)((r \times k), F_0(t)((r \times r)$ are matrices and $z = (z_1^T, z_2^T)^T$. Then the system (19) can be written as follows:

$$\begin{array}{l} F_1(t)z_1 + F_2(t)z_2 = q_1(t), \\ \dot{z}_2 = -F_3(t)z_1 - F_0(t)z_2 - H_0^{-1}q_(t) - B_2\xi, \\ \dot{\xi} = \varphi(\sigma), \quad \sigma = D^T(t)A_1(t)S(t)z - D^T g(t) - Q\xi, \end{array} \tag{22}$$

From Equation (20) it follows that

$$F(t) = T(t)BS(t) + \overline{G}(t), \tag{23}$$

where

$$\overline{G}(t) = D(t)S^{-1}(t)\dot{S}(t) = \left\| \begin{array}{cc} O_1 & O_2 \\ -\dot{C}_3(t) & O_0 \end{array} \right\|, \tag{24}$$

and $O_0(r \times r)$ is the null matrix.

Based on relationships (20), (21) and (23) we derive

$$det\|H(t)\lambda + B(t)\| = detT_{-1}(t)det\|D\lambda + F(t) - \overline{G}\|detS^{-1}(t) =$$

$$= detH_0(t)det\|D\lambda + F(t) - \overline{G}\|. \tag{25}$$

Using Laplace decomposition for computing the determinant from (15), (21), and (25) we obtain

$$det\|D\lambda + F(t) - \overline{G}\| = \lambda^r det F_1(t) + \sum_{i=0}^{r-1} \psi_i(t)\lambda^r, \tag{26}$$

where ψ_i are some functions for $i = 0, \overline{1, \ldots, r-1}$. Therefore, because of (14), (25) and (26) the following inequality is valid:

$$\inf_{t \in I} \det \|F_1(t)\| > 0.$$

Taking into account Expression (22) this inequality implies the equality

$$z_1(t) = -F_1^{-1}(t)F_(t)z_2(t), \tag{27}$$

$$\dot{z}_2(t) = [F_3(t)F_1^{-1}(t)F_2(t) - F_0(t)]z_2(t). \tag{28}$$

Assume that $Z_2(t)$ is the fundamental matrix for solutions of Equation (28). Then from (27) and Definition 2 it follows that the matrix

$$Z(t) = \left\| \begin{matrix} Z_1(t) \\ Z_2(t) \end{matrix} \right\| = \left\| \begin{matrix} F_1^{-1}(t)F_(t)Z_(t) \\ Z_2(t) \end{matrix} \right\|, \tag{29}$$

is fundamental for System (19).

Now we consider System (13) where D and G are defined by the formula (16) with locally summable and bounded on the interval I matrix

$$G_0(t) = F_0(t) - F_3(t)F_1^{-1}(t)F_2(t).$$

If Y is a fundamental matrix of System (13) then there exists a constant matrix $C_0(r \times r)$ for which the following holds:

$$Y(t) = L(t)C_0 = \left\| \begin{matrix} E_1 & F_1^{-1}(t)F_(t) \\ O_3 & E_0 \end{matrix} \right\| \cdot \left\| \begin{matrix} Z_1(t) \\ Z_2(t) \end{matrix} \right\| \cdot C_0. \tag{30}$$

From (19), (21), (23), and (28) it follows that F_1^{-1} and F_2 are absolute continuous matrices that are bounded together with their derivatives in the interval I. Therefore, the matrix L is a Lyapunov matrix. According to Theorem 1, System (22) is asymptotically equivalent to System (11) and, consequently, to System (12).

Now, we note that System (11) may be reduced to the central canonical form. For that, we introduce the operator $L_1(t) = -A(t) - H(t)d/dt$ to System (12).

The following theorem holds.

Theorem 7 (V.P. Yacobets [2]). *Let $A(t)$, $H(t) \in C^{2m}(\alpha,\beta)$, $rank H(t) = k$, and $H(t)$ have the full of Jordan collection with respect to the operator $L_1(t)$ in the interval I which are formed with r cells of degree $l_1, \ldots, l_r$ since $\max_i l_i = m$. Then there exist for all $t \in I$ non-singular $s \times s$-dimensional matrices $\overline{M}G_1(t) \in C^1(\alpha,\beta)$ such that multiplying by $\overline{M}(t)$ and replacing by $x = G_1(t)y$ System (12) is reduced to the following central canonical form*

$$\left\| \begin{matrix} E_{s-r} & 0 \\ 0 & J \end{matrix} \right\| \cdot \dot{y} = \left\| \begin{matrix} M(t) & 0 \\ 0(t) & E_l \end{matrix} \right\| \cdot y + \overline{M}(t)q)(t), \tag{31}$$

where $l = l_1 + l_r$, $J = diag(J_1, \ldots, J_r$ are Jordan cells of degree $l_j, j = 1, ..., r)$.

By Theorem 7 we reduce System (11) to the following system in the central canonical form

$$\begin{aligned} \dot{u} &= -V(t)u - \overline{M_1}(t)q_1 - \overline{M_1}(t)B_9t)\varphi_1(\sigma_1), \\ \dot{J}v &= -v - \overline{M_1}(t)q_1 - \overline{M_1}(t)B_9t)\varphi_1(\sigma_1), \\ \sigma_1 &= Q_1^T u + P_1^T g_1(t) - Q_1\xi_1, \\ \sigma_2 &= Q_2^T u + P_2^T g_1(t) - Q_1\xi_1, \\ \sigma &= (\sigma_1^T, \sigma_2^T)^T, \\ y &= (u^T, v^T)^T. \end{aligned}$$

This system may be investigated with respect to the stability and other quality characteristics when $Q_1(t)$ and $Q_2(t)$ are bounded external perturbations [11], [12].

Next, we present the results of recent research conducted on various qualitative issues of program manifold for differential system, which can be extended to degenerate control systems [13–19].

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Жұматов С.С. БАҒДАРЛАМАЛЫҚ КӨПБЕЙНЕ МАҢАЙЫНДА АЗЫНҒАН ТУРА ЕМЕС БАСҚАРУ ЖҮЙЕЛЕРІН ТҮРЛЕНДІРУ

Айқын емес дифференциалдық жүйелердің бір класы, жоғарғы туынды бойынша шешілмеген жәй дифференциалдыық теңдеулер жүйесі қарастырылады. Мұндай теңдеулер күнделікті өмірде механика, физика, экономика, биология және т.б. салаларда жиі кездеседі. Мұндай теңдеулерге берілген жатық бағдарламалық көпбейне бойынша автоматты басқару жүйелерін құру есебі де келтіріледі. Бұл құрылып жатқан теңдеулер жүйесінің өлшемі бағдарламалық көпбейненің өлшемінен үлкен болғандағы жағдай. Бұл арада тіктөртбұрышты матрицалы алгебралық теңдеулер жүйесі пайда болады. Біз дискриминанты нөлге тең квадратты матрицалы жүйені қарастырамыз. Берілген көпбейне бойынша дифференциалдыық теңдеулер жүйесін құрудың жалпы есебі қарастырылады. Көпбейненің теңдеулер жүйесі үшін интегралдық болуының қажетті және жеткілікті шарттары құрылады. Еругин функциясы көпбейнеге қатысты сызықты етіп таңдап алынады. Сонан соң белгілі бір шарттар орындалғанда берілген көпбейненің жүйе үшін интегралдық болатынын ескере отырып, тура емес басқару жүйесі тұрғызылады. Жалпы жағдайда Якоби матрицасы тіктөртбұрышты болып табылады. Бізде матрицаның квадраттық болуы және нөлдік түбірлері болу жағдайы қарастырылады. Көпбейне ізделінді айнымалыға қатысты сызықты етіп алынады. Жоғарғы туынды бойынша шешілмеген, азынған тура емес басқару жүйесі алынды. Матрицасы тұрақты және арнайы құрылымды белгілі бір жүйеге эквивалентті болуы тағайындалды. Ляпунов түрлендіру матрицасы табылды. Қарастырылып отырған басқару жүйесінің орталық канондық түрге келтіріле алатындығы көрсетілді. Қысқаша шолу жасалынды.

Түйін сөздер: Бағдарламалық көпбейне, азынған жүйелер, жүйенің эквиваленттілігі, тура емес басқару жүйелері, Ляпунов түрлендіруі, канондық түрлер.

Жуматов С.С. ПРЕОБРАЗОВАНИЯ ВЫРОЖДЕННЫХ СИСТЕМ НЕПРЯМЫХ УПРАВЛЕНИЙ В ОКРЕСТНОСТИ ПРОГРАММНОГО МНОГООБРАЗИЯ

Рассматривается один из классов неявных дифференциальных систем, системы обыкновенных дифференциальных уравнений, не разрешенных относительно старшей производной. Такие уравнения часто встречаются в повседневной жизни в механике, физике, экономике, биологии и т.д. К таким уравнениям приводятся и задачи построения систем автоматических управлений по заданному гладкому программному многообразию. Это случай, когда размерность строящихся систем уравнений больше, чем размерность программного многообразия. Тогда возникают системы алгебраических уравнений с прямоугольной матрицей. Мы рассматриваем систему с квадратной матрицей, дискриминант которой равен нулю. Рассматривается общая задача построения систем дифференциальных уравнений по заданному многообразию. Составляются необходимое и достаточное условия того, что многообразие является интегральным для системы уравнений. Выбранная функция Еругина линейна относительно многообразия. Затем строится система непрямого управления с учетом того, что заданное многообразие является интегральным для нее при выполнении некоторых условий. В общем случае матрица Якоби является прямоугольной. Исследуется случай, когда матрица является квадратичной и имеет нулевые корни. Устанавливается эквивалентность к некоторой системе, матрицы которой постоянны и имеют специальную структуру. Найдены матрицы преобразования Ляпунова. Показано, что рассматриваемые системы управления могут быть приведены к центральной канонической форме. Приведен краткий обзор.

Ключевые слова. Программное многообразие, вырожденные системы, эквивалентность систем, автоматические системы непрямого управления, преобразования Ляпунова, канонические формы.