Kazakh Mathematical Journal ISSN 1682–0525

22:1 (2022) 56–73

Generalized solutions of boundary value problems of dynamics of thermoelastic rods

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Communicated by: Stanislav N. Kharin

Received: 23.11.2023 ⋆ Accepted/Published Online: 10.12.2024 ⋆ Final Version: 06.12.2024

Abstract. We consider spatially one-dimensional boundary value problems of uncoupled thermoelasticity, which can be used to study various rod structures under thermal heating conditions. This model effectively describes thermodynamic processes at low strain rates. Here, a unified methodology has been developed for solving various boundary value problems typical of practical applications.
The task is set to determine the thermo-stressed state of a thermoelastic rod under various boundary conditions at its ends, as well as the acting forces and heat sources along the entire length of the rod. Shock elastic waves arising in such structures under the action of shock loads are considered.
Based on the method of generalized functions, generalized solutions of non-stationary and stationary direct and semi-inverse boundary value problems in the class of generalized vector functions of slow growth are constructed. Regular integral representations of generalized solutions are also given, which provide analytical solutions to the posed boundary value problems.
The peculiarity of the constructed solutions makes them convenient for engineering applications because make it possible to study the influence of each boundary condition, as well as the acting power and heat sources separately, which is very important when assessing the strength properties of rod structures.

Keywords. Thermoelasticity, boundary value problems, method of generalized functions, generalized solution, Fourier transform, boundary equations.

1 Introduction

Research on thermoelasticity arose for determining the thermoelastic stress state of various structures. They are based on the theory developed by J. Duhamel [1] and F. Neu-

2010 Mathematics Subject Classification: 35M12.
Funding: This research is funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. BR20281002, 2023–2025).
DOI: https://doi.org/10.70474/k7gyhj30

mann [2], who proceeded from the assumption that total deformation is the sum of elastic deformation based on usual relations of the elasticity theory, and ones corresponding to heat field, known from the classical theory of heat conduction. In the works of M.A. Biot [3], for the first time a complete substantiation of the basic relations and terms were given using the thermodynamics of irreversible processes, and variational theorems were formulated.

In the books by W. Novacki [4, 5], mathematical models are presented in detail for describing thermodynamic processes in deformable solids and media, which motion, temperature and stress-strain state depend on the acting forces and heat sources. He developed various methods for solving the thermoelasticity equations in spaces of different dimensions, and investigated a wide class of solutions for various types of operating forces and heat sources.

The complete system of thermoelasticity equations connects equations of medium motion in displacements and an equation of thermal conductivity into a single system, which takes into account the effect of temperature on stress-strain state, as well as opposite, i.e. the effect of a rate of deformation on the temperature of the medium. If in this system an influence of deformation on temperature is neglected, then its solution is simplified. Since at first, you can determine the temperature field separately, and only then take it into account in the equations of motion as a volume force. This model is called *uncoupled thermoelasticity*, and has received wide practical applications in engineering as *thermal stresses theory*. It describes well thermodynamic processes at low strain rates. A detailed review of works for different models of thermoelasticity is given in the R. Hetnarski encyclopedia [6].

In the works of V.D. Kupradze, T.G. Gegelia [7] and others [8–12], the method of boundary integral equations (BIEM) was developed for solving three-dimensional and two-dimensional BVPs (BVPs) of coupled and uncoupled thermoelasticity using the Laplace or Fourier transform in time to construct resolving boundary integral equations of BVPs. It is not possible to construct such equations in the initial space-time for the equations of coupled thermoelasticity due to the peculiarities of the fundamental solutions, for which analytical formulas can be constructed only for their transformants.

One of the main problems of BIE method in the space of Fourier-Laplace transform, which is quite well known, is the instability of the numerical procedures for inverting the transformants of solutions with increasing time, which does not allow in calculations to construct solutions in the space of originals for even small times for oscillatory processes. Therefore, the problem of constructing the originals of solutions to BVPs of thermoelasticity remains relevant. This problem is solved for the uncoupled thermoelasticity model.

Here, spatially one-dimensional unsteady BVPs of non-coupled thermoelasticity are considered, which can be used to study various bar structures. The unified technique is proposed for solving various BVPs typical of practical applications.

Note that investigations of thermally stress-srtain state of rods with different physical and mechanical properties is engaged by few authors, and mainly on the basis of numerical finite element and difference methods for study of static and quasi-static rods stress-strain state [13, 14]. Earlier, in [15, 16], dynamic BVPs for thermoelastic rods (*coupled thermoelasticity*) under actions of sources of low- and high-frequency oscillations were solved. Here non-stationary and stationary BVPs are considered by actions of forces and heat sources of various types, including those described by singular generalized functions.

2 Formulation of non-stationary BVPs

A thermoelastic rod of a length 2L is considered, which is characterized by a mass density $\rho,$ stiffness EJ and thermoelastic constants γ and κ. The displacements of cross sections of a rod and temperature are described by the system of hyperbolic-parabolic equations of the form [4,5]:

$$\rho c^2 u_{,xx} - \rho u_{,tt} - \gamma \theta_{,x} + \rho F_1 = 0, \tag{1}$$

$$\theta_{,xx} - \kappa^{-1} \theta_{,t} + F_2 = 0. \tag{2}$$

Here $u(x,t)$ are the components of longitudinal displacements in a point x at time t, $\theta(x,t)$ is relative temperature ($\theta = T(x,t) - T_0$), $T(x,t)$ is absolute temperature, T_0 is a temperature in a free state, F_1 is acting force; c is a velocity of thermoelastic waves propagation in a rod. An action of heat sources is described by a function F_2. It is assumed that functions $F_1(x,t)$ and $F_2(x,t)$ belong to the class of generalized functions of slow growth, which makes possibility to simulate thermodynamic processes in rods under actions of concentrated forces and heat sources of various types. Here and further, we use the following notation for partial derivatives: $\partial u/\partial x = \partial_x u$, $\partial u/\partial t = \partial_t u$, $\partial u_i/\partial x = \partial_j u_i = u_{i,j}$, $i = 1,2$, $j = x,t$.

The thermoelastic stress in a rod is determined by the Duhamel–Neumann law:

$$\sigma(x,t) = \rho c^2 u_{,x}(x,t) - \gamma \theta(x,t). \tag{3}$$

Let us consider a number of direct BVPs typical for engineering practice, the solutions of which satisfy the following initial and boundary conditions.

Initial conditions (Cauchy conditions): the displacements, velocities and temperature are known at $t = 0$:

$$u(x,0) = u_0(x), \quad \theta(x,0) = \theta_0(x), \quad |x| \leq L; \tag{4}$$

$$\partial_t u(x,0) = \dot{u}_0(x), \quad |x| < L.$$

Boundary conditions are set at the rod ends ($x = x_1 = -L$, $x = x_2 = L$), which differ depending on the BVPs under consideration. Here we construct solutions of next BVPs (BVP).

BVP 1. The displacements of the rod ends and the temperature on them are known:

$$u(x_j,t) = w_j(t), \quad \theta(x_j,t) = \theta_j(t); \quad j = 1,2. \tag{5}$$

BVP 2. The stresses at the ends of the rod and the heat flows on them are known:

$$\sigma\left(x_j,t\right)=p_j\left(t\right),\quad \theta_{,x}\left(x_j,t\right)=q_j(t);\quad j=1,2. \tag{6}$$

BVP 3. The displacements of the rod ends and the heat flows on them are known:

$$u\left(x_j,t\right)=w_j\left(t\right),\quad \theta_{,x}\left(x_j,t\right)=q_j(t);\quad j=1,2. \tag{7}$$

BVP 4. The stresses and temperature at the ends of the rod are known:

$$\sigma\left(x_j,t\right)=p_j\left(t\right),\quad \theta(x_j,t)=\theta_j(t);\quad j=1,2. \tag{8}$$

BVP 5. The stresses at the ends of the rod, the temperature on the one end and the heat flow on the other are known:

$$\begin{array}{l}\sigma\left(x_j,t\right)=p_j\left(t\right),\quad j=1,2;\\ \theta(x_1,t)=\theta_1(t)\quad \theta_{,x}\left(x_2,t\right)=q_2(t)\end{array} \tag{9}$$

BVP 6. The displacements at the ends of the rod, the temperature on the one end and the heat flow on the other are known:

$$\begin{array}{l}u\left(x_j,t\right)=w_j\left(t\right),\quad j=1,2;\\ \theta(x_1,t)=\theta_1(t)\quad \theta_{,x}\left(x_2,t\right)=q_2(t)\end{array} \tag{10}$$

BVP 7. The displacements and temperature at the left end of the rod, the stresses on the one end and the heat flow at the right end are known:

$$\begin{array}{l}u\left(x_1,t\right)=w_1\left(t\right),\quad \theta(x_1,t)=\theta_1(t);\\ \sigma\left(x_2,t\right)=p_2\left(t\right),\quad \theta_{,x}\left(x_2,t\right)=q_2(t)\end{array} \tag{11}$$

BVP 8. The displacements and heat flow at the left end of the rod, the stresses and temperature at the right end are known:

$$\begin{array}{l}u\left(x_1,t\right)=w_1\left(t\right),\quad \theta_{,x}\left(x_1,t\right)=q_1(t);\\ \sigma\left(x_2,t\right)=p_2\left(t\right),\quad \theta(x_2,t)=\theta_2(t)\end{array} \tag{12}$$

It is assumed that the boundary functions satisfy the next smoothness conditions:

$$u_j(t)\in C(0,\infty),\ \theta_j(t)\in C(0,\infty), q_j(t)\in L_1(0,\infty), p_j(t)\in L_1(0,\infty).$$

In addition to them, also problems will be considered with different number of boundary functions on rod ends.

3 Shock elastic waves

The system of equations (1), (2) has mixed hyperbolic-parabolic type. Due to hyperbolicity, shock waves may occur during shock effects at rod ends.

To derive the conditions on shock wave fronts, we consider solutions of equations (1), (2) in the space of generalized functions. According to the rules of differentiation of regular generalized functions [12, 13] for shock waves, these equations will take the form:

$$\left(\rho c^2 u_{,xx} - \rho u_{,tt} - \gamma\theta_{,x}\right) + \rho F_1 + \left(\left[\rho c^2 u_{,x} - \gamma\theta\right]\nu_x - \rho\left[u_{,t}\right]\nu_t\right)\delta_F(x,t) + \\ + \rho c^2 \partial_x [u]\,\delta_F(x,t) - \rho\partial_t [u]\,\delta_F(x,t) = 0, \quad (13)$$

$$\left(\theta_{,xx} - \kappa^{-1}\theta_{,t}\right) + F_2 + \partial_x\left([\theta]\,\nu_x\delta_F\right) + \left[\theta_{,x}\right]\nu_x\delta_F - \kappa^{-1}[\theta]\,\nu_t\delta_F = 0.$$

Here, square brackets denote the jump of the functions indicated in them on the shock wave fronts, $\delta_F(x,t)$ is a singular generalized function - a simple layer on the wave front, which is a characteristic surface $F \in R^2$. On F

$$c^2\nu_x^2 - \nu_t^2 = 0 \quad \Rightarrow \quad c = -\frac{\nu_t}{\nu_x} \quad (14)$$

where $\nu = (\nu_x, \nu_t)$ is the normal to F in $R^2 = \{(x,t)\}$, c is the velocity of propagation of a shock wave, at which a stresses can have a jump. From equations (14), taking into account (1), it follows:

$$\begin{gathered}\left(\left[\rho c^2 u_{,x} - \gamma\theta\right]_F \nu_x - \rho\left[u_{,t}\right]_F \nu_t\right)\delta_F + \rho c^2\partial_x\left([u]_F\,\delta_F\right) - \rho\partial_t\left([u]_F\,\delta_F\right) = 0, \\ \partial_x\left([\theta]_F\,\nu_x\delta_F\right) + \left(\nu_x\left[\theta_{,x}\right]_F - \kappa^{-1}[\theta]_F\,\nu_t\right)\delta_F = 0.\end{gathered} \quad (15)$$

Since, due to medium continuity, $[u(x,t)]_{F_t} = 0$, and in the region of differentiability, shock waves are generalized solutions of (1), from (14), due to the independence of the singular functions, we obtain the condition of continuity of temperature at a shock wave front: $[\theta(x,t)]_{F_t} = 0$.

Here the wave front has a simple form:

$$F_t = \left\{(x,t) : x \pm ct = x^0\right\}. \quad (16)$$

This is the point of discontinuity of derivatives on the interval $x \in (-L, L)$, which moves at a speed c from the point x^0 where it is formed, in one direction or the other. As a result, it follows from (15) that the following conditions for jumps must be satisfied at shock wave fronts:

$$[u]_{F_t} = 0, \quad [\sigma]_{F_t} = -\rho c\,[\dot{u}]_{F_t}, \quad [\theta]_{F_t} = 0, \quad \left[\theta_{,x}\right]_{F_t} = 0. \quad (17)$$

The first condition for displacements is a necessary condition for preserving of medium continuity. The second condition describes a stress jump (*shock*), which leads to a speed jump at a wave front. It follows from the third and fourth that the temperature and heat flow are continuous at the shock wave front. This distinguishes a coupled thermoelasticity unlike uncoupled thermoelasticity in which the heat flow has a jump at front which are proportional to jump in displacement velocity [19,20]. Here shock waves are purely elastic waves.

The uniqueness of the solution of non-stationary BVPs taking into account shock waves for coupled thermoelasticity model has been shown in [20,21]. Since the model uncoupled thermoelasticity is a special case of this model, the uniqueness of these BVPs follows from there.

4 Generalized solution of non-stationary BVP. Generalized functions method

To determine the solution of the problem, we set a BVP in the space of two-dimensional generalized vector functions of slow growth [17,18]:

$$S_2'(R^2) = \{\hat{f} = (\hat{f}_1(x,t), \hat{f}_2(x,t)), \quad (x,t) \in R^2, \quad \hat{f}_j \in S'(R^2), j = 1,2\}.$$

To do this, we introduce a generalized vector function (we mark them with a cap) which is equal to 0 inside a region of determination:

$$(\hat{u}_1, \hat{u}_2) = \{\hat{u}, \hat{\theta}\} = \{u(x,t)H(L-|x|)H(t), \theta(x,t)H(L-|x|)H(t)\},$$

where u, θ is the classical solution of the BVP under consideration. $H(x)$ is the Heavyside step function, we assume that at the point of discontinuity $H(0) = 0,5$.

Vector-function $(\hat{u}_1, \hat{u}_2)$ satisfies in $S_2'(R^2)$ to the next system of two equations :

$$\begin{aligned} c^2\hat{u}_{,xx} - \hat{u}_{tt} - \tilde{\gamma}c^2\hat{\theta}_{,x} + \hat{F}_1 &= -\{\dot{u}_0(x)\delta(t) + u_0(x)\delta'(t)\}H(L-|x|) + \\ &+ c^2H(t)\{(p_1(t) - \tilde{\gamma}\theta_1(t))\delta(x+L) - \\ &\qquad - (p_2(t)\tilde{\gamma}\theta_2(t))\delta(x-L)\} + \\ &+ c^2H(t)\{w_1(t)\delta'(x+L) - w_2(t)\delta'(x-L)\}, \end{aligned} \tag{18}$$

$$\begin{aligned} \hat{\theta}_{,xx} - \kappa^{-1}\hat{\theta}_{,t} + \hat{F}_2 &= H(t)\,\delta(L+x)\,q_1(t) - H(t)\,\delta(L-x)\,q_2(t) + \\ &+ \theta_1(t)H(t)\,\delta'(L+x) - \theta_2(t)H(t)\,\delta'(L-x) - \\ &- \kappa^{-1}\theta_0(x)\delta(t)\,H(L-|x|)\,. \end{aligned} \tag{19}$$

Here $\delta(t)$ is a generalized singular delta-function, $\tilde{\gamma} = \frac{\gamma}{\rho c^2}$.

Using the property of fundamental solutions to Equations (18) and (19), its solution $\hat{u}, \hat{\theta}$

can be represented as the following convolutions:

$$u\,(x,t)\,H\,(t)\,H\,(L-|x|) \;=\; \hat{U}_1 * \hat{F}_1 - \tilde{\gamma}\hat{U}_{1,x} * \theta(x,t) + \tag{20}$$
$$+c^2\sum_{k=1}^{2}(-1)^k\{(p_k(t)-\tilde{\gamma}\,\theta_k(t)) \underset{t}{*} \hat{U}_{1,x}(x-(-1)^kL,t) +$$
$$+u_k(t) \underset{t}{*} \hat{U}_{1,x}\,(x-(-1)^kL,t)\} +$$
$$+\left\{\dot{u}_0(x)H(L-|x|) \underset{x}{*} \hat{U}_1 + u_0(x)H(L-|x|) \underset{x}{*} \hat{U}_{1,t}\right\},$$

$$\theta\,(x,t)\,H\,(t)\,H\,(L-|x|) = \hat{F}_2(x,t) * \hat{U}_2(x,t) + H\,(t)\sum_{k=1}^{2}(-1)^k\{q_k(t) \underset{t}{*} U_2\left(x-(-1)^kL,t\right) +$$
$$+\,\theta_k(t)H\,(t) \underset{t}{*} U_{2,x}\left(x-(-1)^kL,t\right)\} - \kappa^{-1}\theta_0(x)H\,(L-|x|) \underset{x}{*} U_2(x,t) \tag{21}$$

Here $U_j(x,t)$ (for $j=1,2$) are the fundamental solutions of the wave equation (1) for $F_1=\delta(x,t)=\delta(x)\delta(t)$, $\theta(x,t)=0$, and the heat conduction equation for $F_2=\delta(x,t)=\delta(x)\delta(t)$, which are well known [17,18]:

$$\hat{U}_1(x,t) \;=-\frac{1}{2c}H(ct-|x|)H(t), \tag{22}$$

$$\hat{U}_2(x,t) \;=-\frac{1}{2\kappa\sqrt{\pi t}}\exp(-x^2/4\kappa t)H(t). \tag{23}$$

The integral record of convolutions of the generalized solution has the form:

$$u(x,t)H(L-|x|)H(t) \;=\; \hat{U}_1 * \hat{F}_1 - \tag{24}$$
$$-\tilde{\gamma}H(t)\partial_x\int_0^t d\tau\int_{-L}^{L}U_1(x-y,\tau)\theta(y,t-\tau)dy +$$
$$+c^2H(t)\sum_{k=1}^{2}(-1)^k\int_0^t\{(p_k(\tau)-\tilde{\gamma}\,\theta_k(\tau))\,U_1(x-(-1)^kL,t-\tau) +$$
$$+\,w_k(\tau)U_{1,x}\,(x-(-1)^kL,t-\tau)\}d\tau +$$
$$+H\,(L-|x|)\,H(t)\int_{-L}^{L}\{\dot{u}_0(y)U_1(x-y,t)\}dy -$$
$$-H\,(L-|x|)\,H(t)\partial_t\int_{-L}^{L}\{u_0(y))U_1(x-y,t\}dy,$$

$$\begin{aligned}\theta(x,t)H(t)H(L-|x|) &= \hat{F}_2 * \hat{U}_2 + \\ &+H(t)\sum_{k=1}^{2}(-1)^k \int_0^t \{q_k(\tau)U_2(x-(-1)^k L, t-\tau) + \\ &\qquad + \theta_k(\tau)U_{2,x}(x-(-1)^k L, t-\tau)\}d\tau + \\ &+\kappa^{-1}H(L-|x|)H(t)\int_{-L}^{L}\theta_0(y)U_2(x-y,t)\,dy.\end{aligned} \tag{25}$$

For regular $\hat{F}_j$ the convolution has the form:

$$\hat{F}_j * \hat{U}_j = H(t)\int_0^t \int_{-L}^{L} F_j(y,t-\tau)U_j(x-y,\tau)dyd\tau.$$

For singular $\hat{F}_j$, typical for physical applications, it is necessary to use the definition of the convolution of generalized functions [17, 18].

Formulas (24), (25) determine the displacement and temperature inside the rod by the known displacements, stresses, temperature and heat flows at its ends. However, for correctly posed BVPs, only 4 of the 8 boundary functions are known. To determine the unknown four, it is necessary to construct resolving boundary equations at the left and right ends of the rod. To do this, we consider solutions (24), (25) in the space of the Fourier transform in time. For regular summable functions, the direct and inverse incomplete Fourier transforms in time have the form:

$$\bar{f}(x,\omega) = \int_{-\infty}^{\infty} f(x,t)e^{i\omega t}dt, \quad f(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\bar{f}(x,\omega)e^{-i\omega t}d\omega. \tag{26}$$

For singular functions, we should use the definition of convolution in the space of generalized functions [17, 18].

5 The Fourier transformant in time of Green function of wave equation.

The Green function of wave equation $\hat{U}_1(x,t)$ satisfies the D'Alembert equation:

$$c^2\frac{\partial^2\hat{U}_1}{\partial x^2} - \frac{\partial^2\hat{U}_1}{\partial t^2} + \delta(x)\delta(t) = 0,$$

and radiation conditions:

$$\hat{U}_1(x,t) = 0,\ \ t<0, \quad \hat{U}(x,t) = 0 \text{ for } \|x\| > ct.$$

Its incomplete Fourier transform is the solution of an ordinary differential equation:

$$\frac{d^2\bar{U}_1}{dx^2} + k^2\bar{U}_1 = -c^{-2}\delta(x), \quad k = \frac{\omega}{c}, \tag{27}$$

Here $\delta(x)$ is singular delta-function. It and its derivative have the form:

$$\bar{U}_1(x,\omega) = -\frac{\sin(k\,|x|)}{2c^2(k+i0)}, \qquad \bar{U}_{1,x} = -\frac{1}{2c^2}\cos(k\,|x|)\mathrm{sgn}(x). \tag{28}$$

It is easy to see that

$$\bar{U}_1(0,\omega) = 0, \qquad \bar{U}_{1,x}(\pm 0,\omega) = \mp 0,5c^{-2}. \tag{29}$$

We use this property to construct solving equations of BVPs.

6 Fourier transformant in time of Green function of heat equation

This Fourier transformant satisfies the equation:

$$\frac{d^2\bar{U}_2}{dx^2} + i\,\omega\,\kappa^{-1}\bar{U}_2 + \delta(x) = 0 \tag{30}$$

and the next conditions :

$$\bar{U}_2(x,\omega) = \bar{U}_2(-x,\omega), \quad \bar{U}_2(x,\omega) \underset{|x|\to\infty}{\to} 0.$$

Let denote $k = \sqrt{i\omega\kappa^{-1}} = e^{i\pi/4}\sqrt{\omega\kappa^{-1}} = (1+i)\sqrt{\frac{\omega}{2\kappa}}$. It is not difficult to see that the function

$$\bar{U}_2(x,\omega) = -\frac{\sin(k\,|x|)}{2k} \tag{31}$$

satisfies the equations (30) and

$$\bar{U}_2(0,\omega) = 0, \quad \bar{U}_{2,x}(\pm 0,\omega) = \mp 0,5 \tag{32}$$

To solve the set BVPs, we consider separately generalized solutions of the thermal conductivity equation and the wave equation.

7 Solution of heat BVP in Fourier transform space in time

We define Fourier transform of the rod temperature using its representation (21) and the properties of Fourier transform of convolutions of generalized functions. As a result, we get:

$$\begin{gathered}\bar{\theta}(x,\omega)\,H(L-|x|) = \bar{F}_2(x,\omega) \underset{x}{*} \bar{U}_2(x,\omega) + \\ +\kappa^{-1}\theta_0(x)H(L-|x|) \underset{x}{*} \bar{U}_2(x,\omega) + \\ + \sum_{k=1}^{2}(-1)^k\left\{\bar{q}_k(\omega)\bar{U}_2\left(x-(-1)^kL,\omega\right) + \bar{\theta}_k(\omega)\bar{U}_{2,x}\left(x-(-1)^kL,\omega\right)\right\}\end{gathered} \tag{33}$$

where for regular $F_j(x,t)$:

$$\bar{F}_j(x,\omega) = H(L-|x|)\int_0^\infty F_j(x,t)e^{i\omega t}d\omega \tag{34}$$

(for singular sources, the Fourier transform should be taken according to the rules of the theory of generalized functions).

Let's consider the limit in this formula for $x \to \pm L,\ x \in (-L, L)$:

$$
\begin{aligned}
\lim_{\varepsilon\to+0} \bar{\theta}\,(-L+\varepsilon,\omega) \;=\;& \bar{F}_2(x,\omega) \underset{x}{*} \bar{U}_2(x,\omega)\Big|_{x=-L} + \\
&+\kappa^{-1}\,\theta_0(x)H\,(L-|x|) \underset{x}{*} \bar{U}_2(x,\omega)\Big|_{x=-L} - \\
&-\lim_{\varepsilon\to+0}\{\bar{q}_1(\omega)\bar{U}_2\,(\varepsilon,\omega)+\bar{\theta}_1(\omega)\,\bar{U}_{2,x}\,(\varepsilon,\omega)\} + \\
&+\lim_{\varepsilon\to+0}\left\{\bar{q}_2(\omega)\,\bar{U}_2\,(-2L+\varepsilon,\omega)+\bar{\theta}_2(\omega)\,\bar{U}_{2,x}\,(-2L+\varepsilon,\omega)\right\} \\
\lim_{\varepsilon\to+0} \bar{\theta}(L-\varepsilon,\omega) \;=\;& \bar{F}_2(x,\omega) \underset{x}{*} \bar{U}_2(x,\omega)\Big|_{x=L} + \kappa^{-1}\theta_0(x)H(L-|x|) \underset{x}{*} \bar{U}_2(x,\omega)\Big|_{x=L} - \\
&-\lim_{\varepsilon\to+0}\{\bar{q}_1(\omega)\bar{U}_2(2L-\varepsilon,\omega)+\bar{\theta}_1(\omega)\bar{U}_{2,x}\,(2L-\varepsilon,\omega)\} + \\
&+\lim_{\varepsilon\to+0}\left\{\bar{q}_2(\omega)\bar{U}_2\,(-\varepsilon,\omega)+\bar{\theta}_2(\omega)\bar{U}_{2,x}\,(-\varepsilon,\omega)\right\}
\end{aligned}
$$

Using the property of Fourier transform of the derivative of the fundamental solution (32) from these formulas we obtain boundary equations for determining the desired boundary functions:

$$
\begin{aligned}
0,5\bar{\theta}\,(-L,\omega) \;=\;& \bar{F}_2(x,\omega) \underset{x}{*} \bar{U}_2(x,\omega)\Big|_{x=-L} + \\
&+\kappa^{-1}\,\theta_0(x)H\,(L-|x|) \underset{x}{*} \bar{U}_2(x,\omega)\Big|_{x=-L} + \\
&+\bar{q}_2(\omega)\bar{U}_2\,(-2L,\omega)+\bar{\theta}_2(\omega)\bar{U}_{2,x}\,(-2L,\omega)\,, \\
0,5\bar{\theta}\,(L,\omega) \;=\;& \bar{F}_2(x,\omega) \underset{x}{*} \bar{U}_2(x,\omega)\Big|_{x=L} + \\
&+\kappa^{-1}\,\theta_0(x)H\,(L-|x|) \underset{x}{*} \bar{U}_2(x,\omega)\Big|_{x=L} - \\
&-\bar{q}_1(\omega)\bar{U}_2\,(2L,\omega)-\bar{\theta}_1(\omega)\bar{U}_{2,x}\,(2L,\omega)\,.
\end{aligned}
$$

Hence the next theorem follows.

Theorem 1. *Fourier transformants in time of boundary value functions of BVPs for the heat equation satisfy to the system of linear algebraic equations of the form*:

$$
\begin{gathered}
\left\{\begin{array}{cc} 0,5 & 0 \\ \bar{U}_{2,x}\,(2L,\omega) & \bar{U}_2(2L,\omega) \end{array}\right\}\left\{\begin{array}{c} \bar{\theta}_1\,(\omega) \\ \bar{q}_1(\omega) \end{array}\right\} + \\
+\left\{\begin{array}{cc} -\bar{U}_{2,x}\,(-2L,\omega) & -\bar{U}_2\,(-2L,\omega) \\ 0,5 & 0 \end{array}\right\}\left\{\begin{array}{c} \bar{\theta}_2\,(\omega) \\ \bar{q}_2(\omega) \end{array}\right\} = \left\{\begin{array}{c} \bar{Q}(-L,\omega) \\ \bar{Q}(L,\omega) \end{array}\right\},
\end{gathered}
\tag{35}
$$

where the right-hand sides are determined by the initial conditions and the current heat sources:

$$\bar{Q}(x,\omega) = \bar{F}_2(x,\omega) \underset{x}{*} \bar{U}_2(x,\omega) + \kappa^{-1}\theta_0(x)H\left(L-|x|\right) \underset{x}{*} \bar{U}_2(x,\omega) \tag{36}$$

The resulting system of equations allows us to solve any BVPs for given two boundary functions of temperature and heat flow at the ends of the rod. The other two are determined by the solution of this system.

To solve all the set temperature BVPs, it is convenient to consider an extended system of equations of the form:

$$\begin{Bmatrix} 0,5 & 0 & -\bar{U}_{2,x}\left(-2L,\omega\right) & -\bar{U}_2\left(-2L,\omega\right) \\ \bar{U}_{2,x}\left(2L,\omega\right) & \bar{U}_2\left(2L,\omega\right) & 0,5 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{Bmatrix} \times \\ \times \begin{Bmatrix} \bar{\theta}_1\left(\omega\right) \\ \bar{q}_1(\omega) \\ \bar{\theta}_2\left(\omega\right) \\ \bar{q}_2(\omega) \end{Bmatrix} = \begin{Bmatrix} \bar{Q}(-L,\omega) \\ \bar{Q}(L,\omega) \\ \bar{b}_1(\omega) \\ \bar{b}_2(\omega) \end{Bmatrix} \tag{37}$$

where the last two equations are the boundary conditions at ends of a rod:

$$\begin{Bmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{Bmatrix} \begin{Bmatrix} \bar{\theta}_1\left(\omega\right) \\ \bar{q}_1(\omega) \end{Bmatrix} + \begin{Bmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{Bmatrix} \begin{Bmatrix} \bar{\theta}_2\left(\omega\right) \\ \bar{q}_2(\omega) \end{Bmatrix} = \begin{Bmatrix} \bar{b}_1(\omega) \\ \bar{b}_2(\omega) \end{Bmatrix}. \tag{38}$$

For given coefficients a_{ij} and the right side $\bar{b}_i(\omega)$ of this linear algebraic system of equations, its solution has the form:

$$D_j(\omega) = \frac{\Delta_j(\omega)}{\Delta(\omega)}, \quad D(\omega) = \begin{Bmatrix} \bar{\theta}_1\left(\omega\right) \\ \bar{q}_1(\omega) \\ \bar{\theta}_2\left(\omega\right) \\ \bar{q}_2(\omega) \end{Bmatrix}, \tag{39}$$

where $\Delta(\omega)$is the determinant of the matrix of the system (37), $\Delta_j(\omega)$ the determinant of the matrix, which is determined by a simple Kramer rule for each $D_j(\omega)$.

As an example, we will construct solutions for the temperature field of some BVPs.

8 Temperature solution for BVP1, BVP4.

In this case, the temperature at the ends of the rod is known: $\bar{\theta}(x_j,\omega) = \bar{\theta}_j(\omega), \quad j=1,2.$ Then the boundary conditions (38) have the form:

$$\begin{Bmatrix} 1 & 0 \\ 0 & 0 \end{Bmatrix} \begin{Bmatrix} \bar{\theta}_1\left(\omega\right) \\ \bar{q}_1(\omega) \end{Bmatrix} + \begin{Bmatrix} 0 & 0 \\ 1 & 0 \end{Bmatrix} \begin{Bmatrix} \bar{\theta}_2\left(\omega\right) \\ \bar{q}_2(\omega) \end{Bmatrix} = \begin{Bmatrix} \bar{b}_1(\omega) \\ \bar{b}_2(\omega) \end{Bmatrix} = \begin{Bmatrix} \bar{\theta}_1\left(\omega\right) \\ \bar{\theta}_2\left(\omega\right) \end{Bmatrix}.$$

Substituting these coefficients and the left part of the equations (37), we obtain the solution (39) for determining the heat flow. It has the form:

$$\bar{q}_1(\omega) = \frac{1}{\bar{U}_2(2L,\omega)}\left(\bar{Q}_2(L,x) - \bar{U}_{2,x}(2L,\omega)\bar{\theta}_1(\omega) - 0,5\bar{\theta}_2(\omega)\right)$$
$$\bar{q}_2(\omega) = -\frac{1}{\bar{U}_{2,x}(-2L,\omega)}\left(\bar{Q}_1(-L,x) + \bar{U}_{2,x}(-2L,\omega)\bar{\theta}_2(\omega) - 0,5\bar{\theta}_1(\omega)\right) . \quad (40)$$

8.1 Decision of BVP II.

In the case of BVP II, the heat flow at the ends of the rod are known: $\bar{\theta},_x(x_j,\omega) = \bar{q}_j(\omega); \quad j = 1,2$. The boundary conditions (38) for this problem have the following form:

$$\left\{\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array}\right\}\left\{\begin{array}{c} \bar{\theta}_1(\omega) \\ \bar{q}_1(\omega) \end{array}\right\} + \left\{\begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array}\right\}\left\{\begin{array}{c} \bar{\theta}_2(\omega) \\ \bar{q}_2(\omega) \end{array}\right\} = \left\{\begin{array}{c} \bar{b}_1(\omega) \\ \\ \bar{b}_2(\omega) \end{array}\right\} = \left\{\begin{array}{c} \bar{q}_1(\omega) \\ \bar{q}_2(\omega) \end{array}\right\}.$$

Then we can calculate the unknown temperature:

$$\bar{\theta}_1(\omega) = \frac{\Delta_1}{\Delta}, \quad \bar{\theta}_2(\omega) = \frac{\Delta_2}{\Delta}, \quad (41)$$

$$\Delta = \left|\begin{array}{cc} 0,5 & -\bar{U}_{2,x}(-2L,\omega) \\ \bar{U}_{2,x}(-2L,\omega) & 0,5 \end{array}\right| = 0,25 + \bar{U}_{2,x}(-2L,\omega)\bar{U}_{2,x}(2L,\omega),$$

$$\Delta_1 = \left|\begin{array}{cc} \bar{Q}_1 + \bar{U}_2(-2L,\omega)\bar{q}_2 & -\bar{U}_{2,x}(-2L,\omega) \\ \bar{Q}_2 - \bar{U}_2(2L,\omega)\bar{q}_1 & 0,5 \end{array}\right| =$$

$$= 0,5\left(\bar{Q}_1 + \bar{U}_2(-2L,\omega)\bar{q}_2\right) + \bar{U}_{2,x}(-2L,\omega)\left(\bar{Q}_2 - \bar{U}_2(2L,\omega)\bar{q}_1\right),$$

$$\Delta_2 = \left|\begin{array}{cc} 0,5 & \bar{Q}_1 + \bar{U}_2(-2L,\omega)\bar{q}_2 \\ \bar{U}_{2,x}(2L,\omega) & \bar{Q}_2 - \bar{U}_2(2L,\omega)\bar{q}_1 \end{array}\right| =$$

$$= 0,5\left(\bar{Q}_2 - \bar{U}_2(2L,\omega)\bar{q}_1\right) - \bar{U}_{2,x}(2L,\omega)\left(\bar{Q}_1 + \bar{U}_2(-2L,\omega)\bar{q}_2\right).$$

Here

$$\left\{\begin{array}{c} \bar{Q}_1 \\ \bar{Q}_2 \end{array}\right\} = \left\{\begin{array}{c} \bar{Q}(-L,\omega) \\ \bar{Q}(L,\omega) \end{array}\right\}.$$

8.2 Decision BVP VII

In the case of this boundary value problem, the temperature on the left and the heat flow on the right at the ends of the rod are known. The boundary conditions (38) have the form:

$$\left\{\begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array}\right\}\left\{\begin{array}{c} \bar{\theta}_1(\omega) \\ \bar{q}_1(\omega) \end{array}\right\} + \left\{\begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array}\right\}\left\{\begin{array}{c} \bar{\theta}_2(\omega) \\ \\ \bar{q}_2(\omega) \end{array}\right\} = \left\{\begin{array}{c} \bar{b}_1(\omega) \\ \bar{b}_2(\omega) \end{array}\right\} = \left\{\begin{array}{c} \bar{\theta}_1(\omega) \\ \bar{q}_2(\omega) \end{array}\right\}.$$

The solution has the form:

$$\begin{gathered}\bar{\theta}_2(\omega) = -\tfrac{1}{\bar{U}_{2,x}(-2L,\omega)}\left(\bar{Q}(-L,x) - 0,5\bar{\theta}_1(\omega) + \bar{U}_2(-2L,\omega)\bar{q}_2(\omega)\right),\\ \bar{q}_1(\omega) = \tfrac{1}{\bar{U}_2(2L,\omega)}\left(\bar{Q}(L,x) - \bar{U}_{2,x}(2L,\omega)\bar{\theta}_1(\omega) + 0,5\bar{\theta}_2(\omega)\right).\end{gathered} \tag{42}$$

It is obvious that by setting various coefficients and the right-hand sides of the equations (38), the developed method allows solving problems with boundary conditions of various types, both with local conditions at one and the other ends of the rod, and non-local ones that connect the boundary conditions at its ends. The problem of solvability of BVPs will be determined by the solvability of the extended system of equations (37).

9 Solution of elastic BVP in Fourier transforms space in time

We define the Fourier transform of rod displacements using their representation (20) and the properties of Fourier transform of convolutions of generalized functions. Since the temperature of the rod is determined, it can be considered as an external given force, which, together with the force acting on the rod, we denote by the known initial conditions:

$$\begin{gathered}\hat{\mathrm{P}}(x,t) = \mathrm{P}(x,t)H(t)H(L-|x|) = \hat{U}_1 * \hat{F}_1 - \tilde{\gamma}\,\hat{U}_{1,x} * \hat{\theta}(x,t) + \\ + c^2 \textstyle\sum_{k=1}^{2}(-1)^k \left\{(-\tilde{\gamma}\,\theta_k(t)) \underset{t}{*} \hat{U}_1(x-(-1)^k L,t)\right\} + \\ + \left\{\dot{u}_0(x)H(L-|x|) \underset{x}{*} \hat{U}_1 + u_0(x)H(L-|x|) \underset{x}{*} \hat{U}_{1,t}\right\}.\end{gathered}$$

Its Fourier transformant, respectively, has the form:

$$\begin{gathered}\bar{\mathrm{P}}\,(x,\omega) = \bar{U}_1 * \bar{F}_1 + \{(\dot{u}_0(x) - i\omega\,u_0(x))\,H(L-|x|)\} \underset{x}{*} \bar{U}_1 - \\ -\tilde{\gamma}\,\bar{U}_{1,x} * \bar{\theta}(x,\omega) - c^2 \textstyle\sum_{k=1}^{2}(-1)^k\,\tilde{\gamma}\,\bar{\theta}_k(\omega)\,\bar{U}_1(x-(-1)^k L,\omega).\end{gathered} \tag{43}$$

From formula (20) follows the representation of displacements in the space of Fourier transform in time:

$$\begin{gathered}\bar{u}\,(x,\omega)\,H\,(L-|x|) = \bar{\mathrm{P}}\,(x,\omega) + \\ + c^2 \sum_{j=1}^{2}(-1)^j\{(\bar{p}_j(\omega))\,\overleftarrow{U}_1\,(x-(-1)^j L,\omega) + \\ + \bar{w}_j(\omega)\,\overleftarrow{U}_{1,x}\,(x-(-1)^j L,\omega)\}.\end{gathered} \tag{44}$$

Taking into account the form of Green function of the wave equation (28), we obtain a generalized solution of BVPs in the form:

$$\bar{u}(x,\omega)H(L-|x|) = \left\{\bar{\mathrm{P}}\left(x,\omega\right) - 0,5k^{-1}\bar{p}_1(\omega)\sin(k\left|x+L\right|)+ \right.$$
$$+0,5\bar{p}_2(\omega)k^{-1}\sin(k\left|(x-L)\right|)- \qquad (45)$$
$$\left. -0,5\bar{w}_1(\omega)\cos(k\left|x+L\right|) + 0,5\bar{w}_2(\omega)\cos(k\left|x-L\right|)\right\}.$$

If we pass in this formula to the limit to the left and right edges of the interval, taking into account the property of derivative of Green function (28), we obtain linear algebraic equations for determining unknown boundary functions:

$$0,5\bar{w}_1(\omega) = \left.\bar{\mathrm{P}}\left(x,\omega\right)\right|_{x=-L} + 0,5k^{-1}\bar{p}_2(\omega)\sin(2kL) + 0,5\bar{w}_2(\omega)\cos(2kL),$$

$$0,5\bar{w}_2(\omega) = \left.\bar{\mathrm{P}}\left(x,\omega\right)\right|_{x=L} - 0,5k^{-1}\bar{p}_1(\omega)\sin(2kL) - 0,5\bar{w}_1(\omega)\cos(2kL).$$

We rewrite this system in matrix form.

Theorem 2. *Fourier transformants in time of boundary functions of BVPs for the wave equation*(44) *satisfy to the linear algebraic equations*:

$$\left\{\begin{matrix} 0,5 & 0 \\ 0,5\cos(2kL) & 0,5k^{-1}\sin(2kL) \end{matrix}\right\}\left\{\begin{matrix} \bar{w}_1(\omega) \\ \bar{p}_1(\omega) \end{matrix}\right\}+$$
$$+\left\{\begin{matrix} -0,5\cos(2kL) & -0,5k^{-1}\sin(2kL) \\ 0,5 & 0 \end{matrix}\right\}\left\{\begin{matrix} \bar{w}_2(\omega) \\ \bar{p}_2(\omega) \end{matrix}\right\} = \left\{\begin{matrix} \bar{\mathrm{P}}(-L,\omega) \\ \bar{\mathrm{P}}(L,\omega) \end{matrix}\right\}$$

Depending on the BVP from this system of two linear algebraic equations we obtain the resolving boundary equations for determining the transformants of unknown boundary functions. Let consider some BVPs.

9.1 BVP 1.

The displacements of the ends of the rod are known: $\bar{u}\left(x_j,\omega\right) = \bar{w}_j\left(\omega\right), j = 1,2.$ Then from Theorem 2 we obtain equations for determining boundary displacements:

$$\bar{p}_1(\omega) = \frac{2\bar{\mathrm{P}}_2 - \cos(2kL)\bar{w}_1(\omega) - \bar{w}_2(\omega)}{k^{-1}\sin(2kL)}$$

$$\bar{p}_2(\omega) = -\frac{2\bar{\mathrm{P}}_1 + \cos(2kL)\bar{w}_2(\omega) + \bar{w}_1(\omega)}{k^{-1}\sin(2kL)}$$

9.2. BVP 2. The stresses of the rod ends are known: $\bar{\sigma}(x_j,\omega) = \bar{p}_j(\omega),\ j = 1,\ 2.$, then we get for determining the boundary stresses:

$$\left\{\begin{matrix} 1 & -\cos(2kL) \\ \cos(2kL) & 1 \end{matrix}\right\}\left\{\begin{matrix} \bar{w}_1(\omega) \\ \bar{w}_2(\omega) \end{matrix}\right\} =$$

$$= \left\{ \begin{array}{l} 2P_1 + k^{-1}\sin(2kL)\bar{p}_2(\omega) \\ 2P_2 - k^{-1}\sin(2kL)\bar{p}_1(\omega) \end{array} \right\} = \left\{ \begin{array}{l} \bar{f}_1(\omega) \\ \bar{f}_2(\omega) \end{array} \right\} \quad \Rightarrow$$

$$\bar{w}_j(\omega) = \frac{\Delta_j(\omega)}{\Delta(\omega)}, \quad \Delta(\omega) = 1 + \cos^2(2kL),$$

$$\Delta_1 = \bar{f}_1(\omega) - \bar{f}_2(\omega)\cos(2kL), \quad \Delta_2 = \bar{f}_2(\omega) + \bar{f}_1(\omega)\cos(2kL).$$

Now, in the formula (45), all the boundary functions are defined. A solution in the transformation space has been constructed.

Now in formula (45) all boundary functions are defined. Performing the inverse Fourier transform (26), we obtain the original solution in the original space-time. The construction of the original depends on the type of Fourier transform of the boundary functions and should be considered separately for a particular boundary value problem. A similar procedure was done for solving several boundary value problems for the d'Alembert equation in [22].

Conclusion

Formulas (20) can be used when designing in engineering calculations of bar structures. They make it possible to determine the effect of boundary stresses, displacements, temperature, and heat flows (heating) on the stress-strain state separately for each effect and to evaluate their effect on the structural strength. .

Note that the solution of BVPs in the space of Fourier transforms (without initial conditions) gives a solution to the problems of stationary harmonic oscillations with frequency ω. By action of periodic forces and heat sources in a rod and at its ends, which is typical for engineering and technical problems, the solution can be decomposed into finite or infinite Fourier series:

$$\hat{u}_j(x,t) = \sum_{n=\pm 1,\pm 2\ldots} a_{jn}\hat{u}_{[j]n}(x,\omega_n)\, e^{i\omega_n t}, \quad j = 1,2,$$

where the amplitude of each n-th harmonic of the series is determined by the solution of the corresponding boundary value problem by $\omega = \omega_n$.

The constructed solutions contain solutions of BVPs for the heat equation under various linearly related boundary conditions, which has have independent interest in solving thermal problems.

At $\gamma = 0$ the obtained solutions describe a wide range of problems of the dynamics of an elastic rod (or elastic string), which can also find useful applications in the design of various rod structures made of elastic materials.

As shown in the article, the method of generalized functions allows us to construct solutions of both classical BVPs of thermoelasticity and others with different linearly related boundary conditions, which is very convenient when programming and performing numerical experiments. This feature of the constructed solutions makes them convenient for studying network thermoelastic systems that can be modeled by graphs from thermoelastic rods.

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Алексеева Л.А., Приказчиков Д.А., Дадаева А.Н., Айнакеева Н.Ж. ТЕРМОСЕРПІМДІ ӨЗЕКТЕР ДИНАМИКАСЫНЫҢ ШЕКАРАЛЫҚ ЕСЕПТЕРІНІҢ ЖАЛПЫЛАНҒАН ШЕШІМДЕРІ

Біз термиялық қыздыру жағдайында әртүрлі стержендік құрылымдарды зерттеу үшін пайдаланылуы мүмкін қосылмаған термосерпімділіктің кеңістіктік бір өлшемді шекаралық есептерін қарастырамыз. Бұл модель төмен деформация жылдамдығында термодинамикалық процестерді жақсы сипаттайды. Мұнда практикалық қолданбаларға тән әртүрлі шекаралық есептерді шешудің бірыңғай әдістемесі әзірленді.

Термосерпімді өзекшенің ұштарында әртүрлі шекаралық жағдайларда оның термокернеу күйін, сонымен қатар өзекшенің бүкіл ұзындығы бойынша әсер етуші күштер мен жылу көздерін анықтау міндеті қойылады.

Соққылық жүктемелердің әсерінен мұндай құрылымдарда пайда болатын соққы серпімді толқындар қарастырылады.

Жалпыланған функциялар әдісі негізінде баяу өсудің жалпыланған векторлық функциялары класындағы стационарлы емес және стационарлы тура және жартылай кері шекаралық есептердің жалпыланған шешімдері тұрғызылады. Қойылған шекаралық есептердің аналитикалық шешімдерін қамтамасыз ететін жалпылама шешімдердің тұрақты интегралды бейнелері де берілген.

Құрылған шешімдердің ерекшелігі оларды инженерлік қолдану үшін ыңғайлы етеді, өйткені әрбір шекаралық жағдайдың әсерін, сондай-ақ әсер етуші қуат пен жылу көздерін бөлек зерттеуге мүмкіндік береді, бұл өзек құрылымдарының беріктік қасиеттерін бағалау кезінде өте маңызды.

Түйінді сөздер. Термосерпімділік, шекаралық есептер, жалпыланған функциялар әдісі, жалпылама шешу, Фурье түрлендіруі, шекаралық теңдеулерді шешу.

Алексеева Л.А., Приказчиков Д.А., Дадаева А.Н., Айнакеева Н.Ж. ОБОБЩЕННЫЕ РЕШЕНИЯ КРАЕВЫХ ЗАДАЧ ДИНАМИКИ ТЕРМОУПРУГИХ СТЕРЖНЕЙ

Рассматриваются пространственно-одномерные краевые задачи несвязанной термоупругости, которые можно использовать для исследования различных стержневых конструкций в условиях термического нагрева. Эта модель хорошо описывает термодинамические процессы при малых скоростях деформации. Здесь разработана единая методика для решения различных краевых задач, типичных для практических приложений.

Поставлена задача определения термо-напряженного состояния термоупругого стержня при различных граничных условиях на его концах, а также действующих сил и источников тепла по всей длине стержня.

Рассмотрены ударные упругие волны, возникающие в таких конструкции под действием ударных нагрузок.

На основе метода обобщенных функций построены обобщенные решения нестационарных и стационарных прямых и полуобратных краевых задач в классе обобщенных вектор функций медленного роста. Даны и регулярные интегральные представления обобщенных решений, которые дают аналитические решения поставленных краевых задач.

Особенность построенных решений делает их удобными для инженерных приложений, т.к. позволяют исследовать влияние каждого краевого условия, как и действующих силовых и тепловых источников по отдельности, что очень важно при оценке прочностных свойств стержневых конструкций.

Ключевые слова. Термоупругость, краевые задачи, метод обобщенных функций, обобщенное решение, преобразование Фурье, граничные уравнения.