Kazakh Mathematical Journal ISSN 1682–0525

**22**:1 (2022) 6–14

# Well-posedness of the Tricomi Problem for the Multidimensional Lavrent'ev-Bitsadze Equation

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Communicated by: Muvasharkhan T. Jenaliyev

Received: 09.11.2023 ⋆ Accepted/Published Online: 10.09.2024 ⋆ Final Version: 02.09.2024

Abstract. Numerous applications in physics and engineering involve models with partial differential equations of mixed type. The theory of boundary-value problems for such equations in two dimensions has been well studied. However, the key problem of well-posedness of mixed problems for such equations in multidimensional bounded domains remains unsolved. This paper establishes a mixed domain in which the solution to the Tricomi problem for the multidimensional Lavrent'ev-Bitsadze equation has a unique classical solution.

Keywords. Tricomi problem, mixed domain, classical solution, multidimensional Lavrent'ev-Bitsadze equation, spherical functions.

## 1 Introduction

Many important applications in physics and engineering involve models with partial differential equations of mixed type. For instance, models of electrostatic waves in a cold plasma lead to solving boundary-value problems for mixed elliptic-hyperbolic equations (see [1]).

Another classical application involves modeling vibrations of elastic membranes. Let the membrane deflection be described by a function $u(x,t), x = (x_1, ..., x_m), m \geq 2$. Following Hamilton's principle, one then obtains a multidimensional wave equation. Instead, assuming that in the bending position the membrane lies in equilibrium, we get to the multidimensional Laplace equation. Hence, the process of vibrations of elastic membranes in space is mathematically represented by the multidimensional Lavrent'ev–Bitsadze equation ([2]).

The theory of boundary-value problems for hyperbolic-elliptic equations in two dimensions has been well studied (see, for instance, the monographs [2, 3] and the references therein).

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2010 Mathematics Subject Classification: 35M12.

Funding: This research is funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. BR20281002, 2023–2025).

However, the key problem of well-posedness of mixed problems for such equations in multidimensional bounded domains remains, to the best of our knowledge, currently unsolved ([4]).

A multidimensional version of the Tricomi problem for the Lavrent'ev–Bitsadze equation was initially posed in [4,5] (see also [6]). In the articles [7, 8] the authors prove that this problem in the multidimensional case is ill-posed, i.e., it has multiple solutions.

Naturally then, a key question is: in which mixed domains the solution of the Tricomi problem is well-posed? In this paper, we answer this question by establishing a mixed domain in which the Tricomi problem (for the multidimensional Lavrent'ev–Bitsadze equation) is uniquely solvable; moreover, we obtain an explicit form of the classical solution to this problem.

Let us also mention the article [9], which analyzes the Tricomi problem in a three-dimensional domain.

**2 Statement of the Problem and the Main Result.**

Let $\Omega_\varepsilon$ be a finite domain of the Euclidean space $E_{m+1}$ of points $(x_1, ..., x_m, t)$, bounded for $t > 0$ by the spherical surface $\Gamma : |x|^2+t^2 = 1$, whereas for $t < 0$ by the cones $K_\varepsilon : |x| = -t+\varepsilon$, $K_1 : |x| = 1 + t$, $\frac{\varepsilon-1}{2} \leq t \leq 0$, where $|x|$ is the length of the vector $x = (x_1, \dots, x_m)$, and $0 \leq \varepsilon < 1$.

Let us denote with $\Omega^+$ and $\Omega^-_\varepsilon$ the parts of the domain $\Omega_\varepsilon$, that lie in the half-spaces $t > 0$ and $t < 0$, with $S^\varepsilon$ the general part of the boundaries of the domain $\Omega^+$, $\Omega^-_\varepsilon$ representing the set $\{t = 0,\ \varepsilon < |x| < 1\}$ of the points in $E_m$. The parts of the cones $K_\varepsilon$, $K_1$, bounding the domain $\Omega^-_\varepsilon$, are denoted as $S_\varepsilon$, $S_1$ respectively.

In the domain $\Omega_\varepsilon$ let us consider the multidimensional Lavrent'ev–Bitsadze equation

$$\Delta_x u + (\operatorname{sgn} t)u_{tt} = 0, \tag{1}$$

where $\Delta_x$ is the Laplace operator defined over the variables $x_1, \dots, x_m$, $m \geq 2$.

From here onwards, we conveniently switch from the Cartesian coordinates $x_1, \dots, x_m, t$ to the spherical ones $r, \theta_1, \dots, \theta_{m-1}, t$, $r \geq 0$, $0 \leq \theta_1 < 2\pi$, $0 \leq \theta_i \leq \pi$, $i = 2, \dots, m-1$, $\theta = (\theta_1, ..., \theta_{m-1})$.

The multidimensional Tricomi problem is then the following:

**Problem 1.** Find a solution of the Equation (1) in the domain $\Omega_\varepsilon$ under $t \neq 0$ in the class $C(\overline{\Omega}_\varepsilon) \cap C^2(\Omega^+ \cup \Omega^-_\varepsilon)$ satisfying the following boundary-value conditions:

$$u\Big|_{\Gamma} = \varphi(r, \theta), \tag{2}$$

$$u\Big|_{S_1} = \psi(r, \theta), \tag{3}$$

Let us mention that Problem 1 under $\varepsilon = 0$ in a specific case has been analyzed in [5].

Let $\left\{Y_{n,m}^k(\theta)\right\}$ be a system of linearly independent spherical functions of order $n$, where $1 \leq k \leq k_n$, $(m-2)!n!k_n = (n+m-3)!(2n+m-2)$, and $W_2^l(S)$, for $l = 0, 1, \ldots$, are Sobolev spaces.

The following lemmata hold ([10]).

**Lemma 1.** Let $f(r,\theta) \in W_2^l(S^\varepsilon)$. If $l \geq m-1$, then the series

$$f(r,\theta) = \sum_{n=1}^{\infty} \sum_{k=1}^{k_n} f_n^k(r) Y_{n,m}^k(\theta), \tag{4}$$

as well as the series obtained from it by differentiation of order $p \leq l-m+1$, converges absolutely and uniformly.

**Lemma 2.** For $f(r,\theta) \in W_2^l(S)$, the necessary and sufficient condition is that the coefficients of the series (4) satisfy the inequalities

$$\left|f_0^1(r)\right| \leq c_1, \sum_{n=1}^{\infty} \sum_{k=1}^{k_n} n^{2l} \left|f_n^k(r)\right|^2 \leq c_2, \ \ c_1, \, c_2 = const.$$

We denote by $\varphi_n^k(r)$ and $\psi_n^k(r)$ the coefficients of the decomposition of the series (4) of the functions $\varphi(r,\theta)$ and $\psi(r,\theta)$, respectively.

Let $\varphi(r,\theta) \in W_2^l(\Gamma), \psi(r,\theta) = \left(r - \frac{1+\varepsilon}{2}\right)^{\frac{m+1}{2}} \psi^*(r,\theta)$, $\psi^*(r,\theta) \in W_2^l(S_1)$, $l > \frac{(3m+4)}{2}$.

Then, the following theorem holds.

**Theorem 1.** For any $\varepsilon \geq 0$ Problem 1 is uniquely solvable.

**Proof of Theorem 1.** In spherical coordinates, the equation (1) in the domain $\Omega^+$ has the following form (see [10]):

$$u_{rr} + \frac{m-1}{r} u_r - \frac{1}{r^2} \delta u + u_{tt} = 0, \tag{5}$$

$$\delta \equiv -\sum_{j=1}^{m-1} \frac{1}{g_j \sin^{m-j-1}\theta_j} \frac{\partial}{\partial \theta_j} \left(\sin^{m-j-1}\theta_j \frac{\partial}{\partial \theta_j}\right), \, g_1 = 1, \, g_j = (\sin\theta_1 ... \sin\theta_{j-1})^2, \, j > 1.$$

It is well known that the spectrum of the operator $\delta$ consists of the eigenvalues $\lambda_n = n(n+m-2)$, for $n = 0, 1, \ldots$, to each of which correspond $k_n$ orthonormal eigenfunctions $Y_{n,m}^k(\theta)$.

Given that the desired solution of Problem 1 in the domain $\Omega^+$ belongs to the class $C(\bar{\Omega}^+) \cap C^2(\Omega^+)$, then this solution can be sought in the form

$$u(r,\theta,t) = \sum_{n=0}^{\infty} \sum_{k=1}^{k_n} \bar{u}_n^k(r,t) Y_{n,m}^k(\theta), \tag{6}$$

where $\bar{u}_n^k(r,t)$ are functions to be determined.

Substituting (6) into (5), taking into account the orthogonality of the spherical functions $Y_{n,m}^k(\theta)$ (see [10]), we obtain

$$\bar{u}_{nrr}^k + \frac{m-1}{r}\bar{u}_{nr}^k + \bar{u}_{ntt}^k - \frac{\lambda_n}{r^2}\bar{u}_n^k = 0, \quad k=\overline{1,k_n}, \quad n=0,1,\dots, \tag{7}$$

where we write the boundary condition (2), considering Lemma 1, in the form

$$\bar{u}_n^k(r,\sqrt{1-r^2}) = \varphi_n^k(r),\ k=\overline{1,k_n},\ n=0,1,...,\ 0\le r\le 1. \tag{8}$$

In (7),(8), substituting $\bar{u}_n^k(r,t) = r^{\frac{(1-m)}{2}}u_n^k(r,t)$, and then letting $r=\rho\cos\varphi$, $t=\rho\sin\varphi$, $\rho\ge 0$, $0\le\varphi\le\pi$ we obtain

$$\upsilon_{n\rho\rho}^k + \frac{1}{\rho}\upsilon_{n\rho}^k + \frac{1}{\rho^2}\upsilon_{n\varphi\varphi}^k + \frac{\overline{\lambda}_n}{\rho^2\cos^2\varphi}\upsilon_n^k = 0, \tag{9}$$

$$\upsilon_n^k(1,\varphi) = g_n^k(\varphi), \tag{10}$$

where

$$\upsilon_n^k(\rho,\,\varphi) = u_n^k(\rho\cos\varphi,\,\rho\sin\varphi),\ \overline{\lambda_n} = \frac{[(m-1)(3-m)-4\lambda_n]}{4},$$

$$g_n^k(\varphi) = (\cos\varphi)^{\frac{(m-1)}{2}}\,\varphi_n^k(\cos\varphi),$$

Let us seek the solution of the problem (9)–(10) in the form

$$\upsilon_n^k(\rho,\,\varphi) = R(\rho)\phi(\varphi). \tag{11}$$

Substituting (11) into (9), we obtain

$$\rho^2 R_{\rho\rho} + \rho R_\rho - \mu R = 0, \tag{12}$$

$$\phi_{\varphi\varphi} + \left(\mu + \frac{\bar{\lambda}_n}{\cos^2\varphi}\right)\phi = 0,\ \mu = const. \tag{13}$$

If we look for the solution of the Euler equation (12) in the form $R(\rho)=\rho^s$, $0\le s = const$, we hence obtain $s^2=\mu$.

Henceforth, let's write the equation (13) in the following form:

$$\phi_{\varphi\varphi} = \left[\frac{l(l-1)}{\cos^2\varphi} - s^2\right]\phi,\ l = -n - \frac{(m-3)}{2}. \tag{14}$$

In equation (14), substituting $\xi=\sin^2\varphi$ we obtain the following equation:

$$\xi(\xi-1)g_{\xi\xi} + \left[(\alpha+\beta+1)\xi - \frac{1}{2}\right]g_\xi + \alpha\beta g = 0, \tag{15}$$

$$g(\xi)=\frac{\phi(\varphi)}{\cos^l\varphi},\ \alpha=\frac{(l+s)}{2},\ \beta=\frac{(l-s)}{2}.$$

The general solution of equation (15) can be represented by the formula (see [11]):

$$g_s(\xi)=c_{1s}F\left(\beta,\ \gamma,\ \frac{1}{2};\xi\right)+c_{2s}\sqrt{\xi}F\left(\beta+\frac{1}{2},\ \gamma+\frac{1}{2},\ \frac{3}{2};\xi\right), \tag{16}$$

which is periodic in $\varphi$, if $s=0,1,...,$ where $c_{1s},\ c_{2s}$ are arbitrary independent constants, whereas $F(\beta,\ \gamma,\ \alpha;\ \xi)$ is the Gaussian hypergeometric function.

Consequently, from (11) and (16) it follows that the general solution of the equation (9) can be written as

$$\upsilon^k_{n,\mu}(\rho,\varphi)=\sum_{s=0}^{\infty}\rho^s\cos^l\varphi\left[c_{1s}F\left(\beta,\ \gamma,\ \frac{1}{2};\sin^2\varphi\right)+c_{2s}\sin\varphi F\left(\beta+\frac{1}{2},\gamma+\frac{1}{2},\frac{3}{2};\sin^2\varphi\right)\right]. \tag{17}$$

Because $\left|\upsilon^k_n\left(\rho,\ \frac{\pi}{2}\right)\right|<\infty$, from (17) we obtain

$$c_{1s}F\left(\beta,\ \gamma,\ \frac{1}{2};1\right)+c_{2s}F\left(\beta+\frac{1}{2},\gamma+\frac{3}{2},\frac{3}{2};1\right)=0,$$

or

$$c_{2s}=-\frac{2\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma\left(\frac{1}{2}-\beta\right)\Gamma\left(\frac{1}{2}-\gamma\right)}c_{1s}, \tag{18}$$

where $\Gamma(z)$ is the gamma function.

Substituting (18) into (17), we obtain

$$\upsilon^k_n(\rho,\ \varphi)=\sum_{s=0}^{\infty}c_{1s}\rho^s\cos^l\varphi\left[F\left(\beta,\ \gamma,\ \frac{1}{2};\sin^2\varphi\right)-\right.$$

$$\left.-\frac{2\Gamma(1-\beta)\Gamma(1-\gamma)}{\Gamma\left(\frac{1}{2}-\beta\right)\Gamma\left(\frac{1}{2}-\gamma\right)}\sin\varphi F\left(\beta+\frac{1}{2},\gamma+\frac{1}{2},\ \frac{3}{2};\sin^2\varphi\right)\right]. \tag{19}$$

A well-known result is (see, for instance, [12]) that the system of functions

$$\left\{\frac{1}{2},\ \cos 2s\varphi,\ \sin 2s\varphi,\ s=1,2,...\right\}$$

is complete, orthogonal in $C([0,\pi])$, and therefore it is also closed.

From here, it follows that the function $g^k_n(\varphi)\in C([0,\pi])$ can be decomposed into the series

$$g_n^k(\varphi) = a_{0,n}^k + \sum_{s=1}^{\infty} \left( a_{s,n}^k \cos 2s\varphi + b_{s,n}^k \sin 2s\varphi \right), \tag{20}$$

where

$$a_{0,n}^k = \tfrac{1}{2\pi} \int\limits_0^\pi g_n^k(\varphi) d\varphi, \ a_{s,n}^k = \tfrac{1}{\pi} \int\limits_0^\pi g_n^k(\varphi) \cos 2s\varphi d\varphi,$$
$$b_{s,n}^k = \tfrac{1}{\pi} \int\limits_0^\pi g_n^k(\varphi) \sin 2s\varphi d\varphi, \ s = 1, 2.... \tag{21}$$

Next, subjecting the function (19) to the condition (10), taking into account the decomposition (20) and allowing $\varphi = 0$, we obtain

$$c_{1n} = a_{s,n}^k, \ s = 0, 1, ..., . \tag{22}$$

Therefore, from (6), (19), and (22) it follows that the solution of the problem (5),(8) in the domain $\Omega^+$ is the function

$$u(r,\theta,t) = \sum_{n=0}^{\infty} \sum_{k=1}^{k_n} \sum_{s=p}^{\infty} \Big[ a_{s,n}^k r^{2-m-n} (r^2+t^2)^{\frac{s}{2}+\frac{n}{2}+\frac{(m-3)}{4}}$$
$$F\left( -\frac{n}{2} + \frac{3-m}{4} + \frac{s}{2}, \ -\frac{n}{2} + \frac{3-m}{4} - \frac{s}{2}, \ \frac{1}{2}; \ \frac{t^2}{r^2+t^2} \right) -$$
$$-\frac{2\Gamma\left(1+\frac{n}{2}+\frac{m-3}{4}-\frac{s}{2}\right)\Gamma\left(1+\frac{n}{2}+\frac{m-3}{4}+\frac{s}{2}\right)}{\Gamma\left(\frac{1}{2}+\frac{n}{2}+\frac{m-3}{4}-\frac{s}{2}\right)\Gamma\left(\frac{1}{2}+\frac{n}{2}+\frac{m-3}{4}+\frac{s}{2}\right)} t(r^2+t^2)^{-\frac{1}{2}}$$
$$F\left( -\frac{n}{2} + \frac{5-m}{4} + \frac{s}{2}, \ -\frac{n}{2} + \frac{5-m}{4} - \frac{s}{2}, \ \frac{3}{2}; \ \frac{t^2}{r^2+t^2} \right) \Big] Y_{n,m}^k(\theta). \tag{23}$$

From (23), for $t \longrightarrow +0$ we obtain

$$u(r,\theta,0) = \tau(r,\theta) = \sum_{n=0}^{\infty} \sum_{k=1}^{k_n} \sum_{s=p}^{\infty} a_{s,n}^k r^{s+\frac{1-m}{2}} Y_{n,m}^k(\theta), \tag{24}$$

where $p \geq \frac{m-1}{2}$, whereas $a_{s,n}^k$ are determined from (21).

It is known that if $g_n^k(\varphi) \in C^q((0,\pi))$, then the following estimate takes place (see [13]) $|a_{s,n}^k| \leq \frac{c_1}{s^{q+2}}$, $q = 0, 1, ...$, and, moreover, the following formulae hold (see [14]):

$$\frac{d^q}{dz^q} F(a,b,c;z) = \frac{(a)_q (b)_q}{(c)_q} F(a+q, \ b+q, \ c+q; \ z), \ \ q = 0, 1...,$$

$$(a)_q = \frac{\Gamma(a+q)}{\Gamma(a)},\ \frac{\Gamma(z+\alpha)}{\Gamma(z+\beta)} = z^{\alpha-\beta}\left[1+\frac{1}{2z}(\alpha-\beta)(\alpha-\beta-1)+O(z^{-2})\right],$$

as well as the following estimates hold (see [10]):

$$|k_n| \le c_1 n^{m-2},\ \Big|\frac{\partial^q}{\partial\theta_j^q}Y_{n,m}^k(\theta)\Big| \le c_1 n^{\frac{m}{2}-1+q},\ j=\overline{1,\,m-1},\ q=0,1...,.$$

From the embedding theorem (see [15]) it follows that $W_2^l(S^\varepsilon) \subset C^q(S^\varepsilon) \cap C(\bar{S}^\varepsilon)$ provided that $l > q + \frac{m}{2}$.

The above analysis, together with the lemmata and the boundary-value conditions, imply that the solution of the form of equation (23) $u(r,\theta,t) \in C(\bar{\Omega}^+) \cap C^2(\Omega^+)$, where

$$\tau(r,\theta) = r^2\tau^*(r,\theta),\ \tau^*(r,\theta) \in W_2^l(S^\varepsilon),\ l > \frac{3m}{2}+2.$$

Hence, taking into account the boundary conditions (3) and (24), we arrive, in the domain $\Omega_\varepsilon^-$ to the Darboux-Protter problem for the multidimensional wave equation:

$$\Delta_x u - u_{tt} = 0 \tag{25}$$

with the following conditions:

$$u\Big|_{S^\varepsilon} = \tau(r,\theta),\ u\Big|_{S_1} = \psi(r,\theta) \tag{26}$$

for which the following theorem has been established (see [16-18]):

**Theorem 2.** For $\varepsilon \ge 0$ the problem (25)–(26) has the unique solution.

Next, using Theorem 2, we arrive to the validity of Theorem 1.

Given that in [16–18] the explicit form of the classical solution of the problem (25)–(26) has been obtained, then we can also analogously derive an explicit representation also for the solution of Problem 1.

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Алдашев С.А. КӨПӨЛШЕМДІ ЛАВРЕНТЬЕВ-БИЦАДЗЕ ТЕҢДЕУІ ҮШІН ТРИКОМИ ЕСЕБІНІҢ КОРРЕКТІЛІГІ

Физика мен техникадағы көптеген қосымшалар аралас типті ішінара дифференциалдық теңдеулері бар модельдерді қамтиды. Екі өлшемдегі мұндай теңдеулер үшін шекаралық есептердің теориясы жақсы зерттелген. Дегенмен, көпөлшемді шектелген облыстардағы мұндай теңдеулер үшін аралас есептерді дұрыс қоюдың негізгі мәселесі қазіргі уақытта шешілмеген күйінде қалып отыр. Бұл жұмыста аралас облыс келтірілген, онда көпөлшемді Лаврентьев-Бицадзе теңдеуіне Трикоми есебінің классикалық жалғыз шешімі бар екендігі дәлелденген.

*Түйінді сөздер.* Трикоми есебі, аралас облыс, классикалық шешім, көпөлшемді Лаврентьев-Бицадзе теңдеуі, сфералық функциялар.

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Алдашев С.А. КОРРЕКТНОСТЬ ЗАДАЧИ ТРИКОМИ ДЛЯ МНОГОМЕРНОГО УРАВНЕНИЯ ЛАВРЕНТЬЕВА-БИЦАДЗЕ

Многочисленные приложения в физике и технике включают модели с уравнениями в частных производных смешанного типа. Теория краевых задач для таких уравнений в двумерном пространстве хорошо изучена. Однако ключевая проблема корректности смешанных задач для таких уравнений в многомерных ограниченных областях остается в настоящее время нерешенной. В данной работе установлена смешанная область, в которой решение задачи Трикоми для многомерного уравнения Лаврентьева-Бицадзе имеет единственное классическое решение.

*Ключевые слова.* Задача Трикоми, смешанная область, классическое решение, многомерное уравнение Лаврентьева-Бицадзе, сферические функции.