Kazakh Mathematical Journal ISSN 1682–0525

23:3 (2023) 6–17

Boundary value problems in time-varying linear differential-algebraic equations: a standard canonical form approach to solvability

Roza E. Uteshova

Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan

r.uteshova@math.kz

Communicated by: Anar T. Assanova

Received: 31.05.2025 ⋆ Accepted/Published Online: 10.06.2025 ⋆ Final Version: 31.05.2025

Abstract. This paper addresses the solvability of two-point boundary value problems for linear differential-algebraic equations with time-varying coefficients. The proposed method employs the standard canonical form to decouple the system into an ordinary differential part and an algebraic part. By introducing an appropriate parameter, we transform the original problem into the solvability of an associated linear algebraic system. This reduction leads to a constructive solvability criterion for the boundary value problem. A comprehensive example is provided to demonstrate the applicability and effectiveness of the proposed approach.

Keywords: Boundary value problem, time-varying linear differential-algebraic equation, standard canonical form, generalized inverse

1 Introduction

We consider a system of linear differential-algebraic equations with time-varying coefficients, given by

$$E(t)\dot{x}(t) = A(t)x(t) + f(t), \quad t \in (0, T), \tag{1}$$

subject to the boundary condition

$$Bx(0) + Cx(T) = d, \tag{2}$$

2020 Mathematics Subject Classification: 34A09; 34B05.

Funding: This research is funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP19675193).

DOI: https://doi.org/10.70474/w14jvq28

where $E, A \in C([0,T];\mathbb{R}^{n\times n})$ are given matrix-valued functions, $f(t) \in C([0,T],\mathbb{R}^n)$ is a given forcing term, and $B, C \in \mathbb{R}^{l\times n}$, $d \in \mathbb{R}^l$ specify the boundary constraints; $T > 0$. Since the matrix $E(t)$ may be singular for all $t \in [0,T]$, equation (1) constitutes a differential-algebraic system rather than a standard ordinary differential equation.

A function $x \in C^1([0,T],\mathbb{R}^n)$ is called a solution of the boundary value problem (1)–(2) if it satisfies the differential-algebraic equation (1) and the boundary condition (2).

Differential-algebraic equations (DAEs) play a central role in the modeling and numerical simulation of constrained dynamical systems across numerous scientific and engineering domains (see, e.g., [8, 9, 11]).

Two-point boundary value problems for DAEs arise naturally in practical settings, including the simulation of electrical circuits and mechanical systems with multiple interconnected components. Initial progress in this area was driven by extending classical techniques — such as shooting and collocation methods — originally developed for ordinary differential equations, to the DAE context [1, 2, 7, 12]. A comprehensive summary of these developments can be found in [10].

The framework proposed in [10] provides a broad foundation capable of handling nonlinear problems as well as time-dependent and operator-valued coefficients, where the coefficient matrices $E(t)$ and $A(t)$ are understood as linear operators in Banach spaces. This line of research is grounded in a projector-based methodology, as systematically developed in [9].

The standard canonical form (SCF) for time-varying DAEs was introduced by Campbell [5, 6] as a structural tool to decouple a DAE into an ordinary differential part and a purely algebraic part involving a pointwise strictly lower triangular matrix. Building upon this foundation, Berger and Ilchmann [4] developed a detailed solution theory for time-varying DAEs that can be transformed into SCF. Their contributions include a rigorous definition of SCF as a canonical form, the derivation of variation-of-constants formulas for inhomogeneous systems, and the demonstration that SCF-transformability is equivalent to analytic solvability. Furthermore, they established connections between SCF and other approaches to the analysis of time-varying DAEs, including the differentiation index, the derivative array method, and the strangeness index. An additional practical outcome of their work is an algorithm for computing the transformation matrices that bring a DAE into SCF.

In the present work, we apply the standard canonical form to investigate the solvability of two-point boundary value problems for time-varying linear DAEs of the form (1)–(2). By transforming the system into SCF, we obtain a decoupled representation that separates the differential and algebraic components of the solution. To treat the boundary conditions, we introduce a parameter that represents the initial value of the differential part of the solution at $t = 0$. This leads to a parametrized initial value problem for the differential part and an explicit representation of the algebraic component. Substituting the resulting expressions into the boundary condition reduces the original boundary value problem to a system of linear algebraic equations in terms of the introduced parameter. Then, a solvability criterion is derived by analyzing this algebraic system.

The structure of the paper is as follows. In Section 2, we review the standard canonical form for time-varying linear DAEs and explain how it decouples the system into dynamic and algebraic components. We also recall the concept of the generalized inverse and its application to the solution of linear algebraic systems. Section 3 contains the main result: we apply the SCF in combination with a parameterization technique to derive a solvability criterion for the boundary value problem and provide an explicit representation of the solution. In Section 4, we demonstrate the applicability of the method through an illustrative example, in which we explicitly solve a boundary value problem and verify the conditions of the main theorem.

2 Preliminaries

2.1. Standard canonical form for linear DAEs

We consider the homogeneous system associated with the inhomogeneous differential-algebraic equation (1), given by

$$E(t)\dot{x}(t) = A(t)x(t), \tag{3}$$

where $E(t), A(t) \in C(\mathcal{I}; \mathbb{R}^{n\times n})$ are matrix-valued functions defined on an open interval $\mathcal{I} \subset \mathbb{R}$.

A function $x : \mathcal{J} \to \mathbb{R}^n$, with $\mathcal{J} \subseteq \mathcal{I}$, is called a *solution* of equation (3) if it is continuously differentiable on $\mathcal{J}$ and satisfies the equation for all $t \in \mathcal{J}$. If $\mathcal{J} = \mathcal{I}$, the solution is called a *global solution.* For brevity, we identify the matrix pair (E, A) with the DAE (3).

It is known that if $(V, W) \in C(\mathcal{I}; \mathbf{Gl}_n(\mathbb{R})) \times C^1(\mathcal{I}; \mathbf{Gl}_n(\mathbb{R}))$, then $x : \mathcal{J} \to \mathbb{R}^n$ solves (3) if, and only if, $z(t) := W(t)^{-1}x(t)$ solves

$$V(t)E(t)W(t)\dot{z} = \big[V(t)A(t)W(t) - V(t)E(t)\dot{W}(t)\big]z.$$

In what follows, $\mathbf{Gl}_n(\mathbb{R})$ denotes the general linear group of degree n, i.e. the set of all invertible $n \times n$ matrices over $\mathbb{R}$.

Definition 1. [8, Def. 3.3] The DAEs (E_1, A_1), $(E_2, A_2) \in C(\mathcal{I}; \mathbb{R}^{n\times n})^2$ are said to be *equivalent* if there exist

$$(V, W) \in C(\mathcal{I}; \mathbf{Gl}_n(\mathbb{R})) \times C^1(\mathcal{I}; \mathbf{Gl}_n(\mathbb{R}))$$

such that

$$E_2 = VE_1W, \quad A_2 = VA_1W - VE_1\dot{W},$$

in which case, we write

$$(E_1, A_1) \overset{V,W}{\sim} (E_2, A_2).$$

This equivalence preserves the solution structure of the DAEs and provides a natural framework for transforming systems into canonical forms.

Definition 2 (Standard canonical form [5])**.** The DAE $(E, A) \in C(\mathcal{I}; \mathbb{R}^{n\times n})^2$ is said to be *transferrable into standard canonical form* if and only if there exist

$$(V, W) \in C(\mathcal{I}; \mathbf{Gl}_n(\mathbb{R})) \times C^1(\mathcal{I}; \mathbf{Gl}_n(\mathbb{R}))$$

and integers $n_1, n_2 \in \mathbb{N}_0$ such that

$$(E, A) \overset{V,W}{\sim} \left(\begin{bmatrix} I_{n_1} & 0 \\ 0 & N \end{bmatrix}, \begin{bmatrix} J & 0 \\ 0 & I_{n_2} \end{bmatrix} \right), \tag{4}$$

where $N : \mathcal{I} \to \mathbb{R}^{n_2\times n_2}$ is pointwise strictly lower triangular, and $J : \mathcal{I} \to \mathbb{R}^{n_1\times n_1}$.

Recall that a matrix $N(t)$ is called *pointwise strictly lower triangular* if all entries on and above the main diagonal vanish for all $t \in \mathcal{I}$.

The standard canonical form separates the differential and algebraic components of the DAE in a structurally transparent way. In the transformed system (4), the upper block corresponds to an ordinary differential equation involving the matrix $J(t)$, while the lower block represents a purely algebraic constraint defined by the pointwise strictly lower triangular matrix $N(t)$. This decoupling is particularly useful for analyzing qualitative properties of the system, such as solvability and stability, and serves as the foundation for the parameterization method employed in this paper.

2.2. Generalized inverse: definitions and application to linear algebraic systems

Definition 3. Let $A \in \mathbb{R}^{m\times n}$ be a given matrix. A matrix $A^g \in \mathbb{R}^{n\times m}$ is called a *generalized inverse* of A if it satisfies

$$AA^gA = A.$$

It is well known that such an inverse always exists, regardless of the rank or dimensions of A. This notion is sometimes referred to as a $\{1\}$-inverse, since it fulfills only the first of the four Penrose conditions used in defining the Moore–Penrose pseudoinverse (see [3]). Importantly, the generalized inverse is not unique in general.

Generalized inverses are a powerful tool for analyzing the solvability of linear systems. Given a system

$$Ax = b,$$

with $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, the system has at least one solution if and only if there exists a generalized inverse A^g such that

$$AA^gb = b. \tag{5}$$

In that case, the complete set of solutions is given by

$$x = A^gb + (I_n - A^gA)z, \tag{6}$$

where $z \in \mathbb{R}^n$ is arbitrary. Condition (5) is equivalent to requiring that $b \in \operatorname{ran}(A)$.

When A is square and invertible, its unique generalized inverse coincides with the standard inverse A^{-1}.

We also make use of special cases of generalized inverses: the left and right inverses, which are applicable to full-rank rectangular matrices.

Definition 4. A matrix $A_L^{-1} \in \mathbb{R}^{n\times m}$ is called a *left inverse* of A if

$$A_L^{-1}A = I_n,$$

where I_n is the $n \times n$ identity matrix.

A left inverse exists if and only if A has full column rank, i.e., $\operatorname{rank}(A) = n$. One example is the matrix $(A^\top A)^{-1}A^\top$, which serves as a left inverse when A has full column rank.

Definition 5. A matrix $A_R^{-1} \in \mathbb{R}^{n\times m}$ is called a *right inverse* of A if

$$AA_R^{-1} = I_m,$$

where I_m is the $m \times m$ identity matrix.

A right inverse exists if and only if A has full row rank, i.e., $\operatorname{rank}(A) = m$. In this case, the matrix $A^\top(AA^\top)^{-1}$ is an example of a right inverse.

3 A solvability criterion for the boundary value problem (1)–(2)

In this section, we analyze the boundary value problem (1)–(2) for a time-varying differential-algebraic system with two-point boundary conditions. We assume that the DAE is transferable into the standard canonical form on the interval $[0, T]$. Under this assumption, the system can be transformed into a decoupled form consisting of an ordinary differential equation and a purely algebraic equation, as described in Section 2.1. This structure enables us to reduce the boundary value problem to a parametric initial value problem and a system of linear algebraic equations involving a suitable parameter. Based on this reduction, we formulate a solvability criterion in terms of the consistency of the associated algebraic system.

Suppose that the DAE $(E, A) \in C([0, T]; \mathbb{R}^{n\times n})^2$ is transferable into SCF via some pair of matrix functions $(V, W) \in C([0, T]; \mathrm{Gl}_n(\mathbb{R}))^2$. This means that, under the transformation

$$x(t) = W(t)y(t) = W(t)\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \tag{7}$$

where the splitting corresponds to the block sizes n_1 and n_2 in the SCF (4), and after multiplying equation (1) from the left by $V(t)$, the system transforms into

$$\dot{y}_1(t) = J(t)y_1(t) + f_1(t), \tag{8a}$$

$$N(t)\dot{y}_2(t) = y_2(t) + f_2(t), \tag{8b}$$

with

$$V(t)f(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix},$$

where $f_1 \in C([0,T];\mathbb{R}^{n_1})$, $f_2 \in C([0,T];\mathbb{R}^{n_2})$.

The transformation (7) also induces a decomposition of the boundary condition matrices:

$$\widetilde{B} := BW(0) = (\widetilde{B}_1, \widetilde{B}_2), \quad \widetilde{C} := CW(T) = (\widetilde{C}_1, \widetilde{C}_2),$$

where $\widetilde{B}_1, \widetilde{C}_1 \in \mathbb{R}^{l\times n_1}$ and $\widetilde{B}_2, \widetilde{C}_2 \in \mathbb{R}^{l\times n_2}$. Hence, the boundary condition (2) becomes

$$\widetilde{B}_1 y_1(0) + \widetilde{B}_2 y_2(0) + \widetilde{C}_1 y_1(T) + \widetilde{C}_2 y_2(T) = d. \tag{9}$$

The decoupled system (8) consists of an ordinary differential equation (8a) for the component y_1, and a pure DAE (8b) for $y_2(t)$. To analyze the boundary value problem, we aim to express the general solutions of both subsystems and substitute them into the transformed boundary condition (9).

Since the algebraic subsystem (8b) does not involve derivatives of $y_2(t)$ in an invertible manner, it does not require initial conditions for its solution. Under the assumption that $f_2(t) \in C^{n_2}([0,T];\mathbb{R}^{n_2})$, the solution can be written explicitly in terms of $f_2(t)$ and its derivatives (see, e.g., [4]) as

$$y_2(t) = -\sum_{k=0}^{n_2-1} \left(N(t)\frac{d}{dt}\right)^k f_2(t). \tag{10}$$

In contrast, the differential subsystem (8a) constitutes an initial value problem, which requires specification of an initial condition at $t = 0$. To this end, we introduce a parameter

$$\lambda := y_1(0) \in \mathbb{R}^{n_1}$$

and define the function

$$z(t) := y_1(t) - \lambda.$$

The function $z(t)$ then satisfies the following initial value problem:

$$\dot{z} = J(t)[z(t) + \lambda] + f_1(t), \quad z(0) = 0. \tag{11}$$

For fixed λ, the solution of the initial value problem (11) can be expressed using the variation of constants formula. Let $\Phi(t,s)$ denote the evolution operator (or transition matrix) associated with the homogeneous system

$$\dot{z} = J(t)z,$$

satisfying

$$\frac{d}{dt}\Phi(t,s) = J(t)\Phi(t,s), \quad \Phi(s,s) = I_{n_1}.$$

Then the solution $z(t)$ of (11) is given by

$$z(t) = \int_0^t \Phi(t,\tau)\left[J(\tau)\lambda + f_1(\tau)\right]d\tau. \tag{12}$$

Recalling that $y_1(t) = z(t) + \lambda$, we obtain the following representation of the solution $y_1(t)$:

$$y_1(t) = \left[I_{n_1} + \int_0^t \Phi(t,\tau)J(\tau)d\tau\right]\lambda + \int_0^t \Phi(t,\tau)f_1(\tau)d\tau. \tag{13}$$

Substituting the expressions for $y_1(0) = \lambda$, $y_1(T)$ from (13), and $y_2(0)$, $y_2(T)$ from (10) into the transformed boundary condition (9), we obtain a linear algebraic system for the unknown parameter $\lambda \in \mathbb{R}^{n_1}$:

$$Q\ \lambda = \widehat{d}, \tag{14}$$

where

$$\begin{aligned} Q :=& \widetilde{B}_1 + \widetilde{C}_1\Big(I_{n_1} + \int_0^T \Phi(T,\tau)J(\tau)d\tau\Big) \in \mathbb{R}^{l\times n_1},\\ \widehat{d} :=& d - \widetilde{C}_1\Big(I_{n_1} + \int_0^T \Phi(T,\tau)f_1(\tau)d\tau\Big)\\ &+\ \widetilde{B}_2\left[\sum_{k=0}^{n_2-1}\left(N(t)\frac{d}{dt}\right)^k f_2(t)\right]_{t=0} + \widetilde{C}_2\left[\sum_{k=0}^{n_2-1}\left(N(t)\frac{d}{dt}\right)^k f_2(t)\right]_{t=T} \in \mathbb{R}^l. \end{aligned} \tag{15}$$

This is a linear system in the unknown λ, whose solvability determines whether the original boundary value problem admits a solution. In the next step, we will formulate the solvability criterion based on this algebraic system. As discussed in Section 2, equation (14) has a solution if and only if $\widehat{d} \in \operatorname{ran}(Q)$. If this condition is fulfilled, we determine λ by

$$\lambda = Q^g\widehat{d} + (I_{n_1} - Q^gQ)c$$

where $c \in \mathbb{R}^{n_1}$, substitute it into the solution formula (13), and combine it with (10). Applying the inverse transformation $x(t) = W(t)y(t)$, we then obtain the general solution of the boundary value problem (1)–(2).

We are now in a position to state the main result.

Theorem 6. *Suppose that:*

(i) the DAE $(E,A) \in C([0,T];\mathbb{R}^{n\times n})^2$ is transferable into SCF via some pair $(V,W) \in C([0,T];\mathrm{Gl}_n(\mathbb{R}))^2$;

(ii) $f_1 \in C([0,T];\mathbb{R}^{n_1})$, $f_2 \in C^{n_2}([0,T];\mathbb{R}^{n_2})$.

Then the boundary value problem (1)–(2) *admits a solution* $x \in C^1([0,T];\mathbb{R}^n)$ *if and only if*

$$\widehat{d} \in \operatorname{ran}(Q). \tag{16}$$

In this case, every solution $x(t)$ *is given by*

$$x(t) = W(t)\begin{bmatrix} \left(I_{n_1} + \int_0^t \Phi(t,\tau)J(\tau)\,d\tau\right)\left(Q^g\widehat{d} + (I_{n_1} - Q^gQ)c\right) + \int_0^t \Phi(t,\tau)f_1(\tau)\,d\tau \\ -\sum\limits_{k=0}^{n_2-1}\left(N(t)\frac{d}{dt}\right)^k f_2(t) \end{bmatrix}, \tag{17}$$

where $c \in \mathbb{R}^{n_1}$ *is an arbitrary vector and* Q^g *is a generalized inverse of* Q*.*

4 An illustrative example

In this section, we apply Theorem 6 to a concrete boundary value problem based on Example 5.6 from [4]. In that work, an algorithm was proposed for transforming real analytic matrix pairs (E, A) into SCF. Rather than carrying out the transformation ourselves, we utilize the SCF of a DAE that has already been computed in the cited example. Our aim is to verify the solvability of the corresponding boundary value problem and to explicitly construct its solution.

Consider the time-varying DAE of the form

$$E(t)\dot{x}(t) = A(t)x(t) + f(t), \quad t \in [0,T],$$

together with the boundary condition

$$Bx(0) + Cx(T) = d.$$

We define the coefficient matrices and right-hand side as follows:

$$E(t) = \begin{bmatrix} \sin t & \cos t & 0 \\ 0 & 0 & 0 \\ -\cos t \sin t & \sin^2 t & 0 \end{bmatrix},$$

$$A(t) = \begin{bmatrix} \sin t - \cos t & \cos t + \sin t & 0 \\ -\cos t & \sin t & 0 \\ -\sin^2 t & -\sin t \cos t & t^2 + 1 \end{bmatrix},$$

$$f(t) = \begin{bmatrix} -1 \\ 0 \\ -t^3 - t \end{bmatrix}, \quad B = I_3, \quad C = \begin{bmatrix} 1 & 0 & 1/\pi \\ 2 & 1 & 0 \\ 0 & \pi & 1 \end{bmatrix}, \quad d = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad T = \pi.$$

As shown in [4], the system is transferable into SCF via the transformation pair $(V(t), W(t))$, with

$$\big(E(t), A(t)\big) \overset{V,W}{\sim} \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \dfrac{\sin t}{t^2+1} & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right),$$

where

$$V(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \dfrac{1}{t^2+1} \end{bmatrix},$$

$$W(t) = \begin{bmatrix} \sin t & -\cos t & 0 \\ \cos t & \sin t & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Thus, in SCF, the system is characterized by the matrices

$$J(t) = 1, \quad N(t) = \begin{bmatrix} 0 & 0 \\ \dfrac{\sin t}{t^2+1} & 0 \end{bmatrix}, \quad n_1 = 1, \quad n_2 = 2.$$

We now proceed to apply Theorem 6 to verify the solvability of the boundary value problem and to construct its explicit solution.

We first compute the transformed inhomogeneity:

$$V(t)f(t) = \begin{bmatrix} -1 \\ 0 \\ -t \end{bmatrix}, \quad f_1(t) = -1, \quad f_2(t) = \begin{bmatrix} 0 \\ -t \end{bmatrix}.$$

The boundary matrices transform as follows:

$$\widetilde{B} = BW(0) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \widetilde{B}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \widetilde{B}_2 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix};$$

$$\widetilde{C} = CW(\pi) = \begin{bmatrix} 0 & 1 & \frac{1}{\pi} \\ -1 & 2 & 0 \\ -\pi & 0 & 1 \end{bmatrix}, \quad \widetilde{C}_1 = \begin{bmatrix} 0 \\ -1 \\ -\pi \end{bmatrix}, \quad \widetilde{C}_2 = \begin{bmatrix} 1 & \frac{1}{\pi} \\ 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

It is clear that assumptions (i) and (ii) of Theorem 6 are satisfied. We now verify the solvability condition (14), which requires computing the matrix Q and the right-hand side $\widehat{d}$ from equation (15). These expressions involve the transition matrix $\Phi(t,s)$ associated with the homogeneous differential equation $\dot{z} = J(t)z$, which in this case reduces to $\dot{z} = z$. Therefore, $\Phi(t,s) = e^{t-s}$.

Using this, we compute

$$Q = \widetilde{B}_1 + \widetilde{C}_1 \left(1 + \int_0^{\pi} e^{\pi-\tau}\, d\tau\right) = \begin{bmatrix} 0 \\ 1-e^{\pi} \\ -\pi e^{\pi} \end{bmatrix}$$

and

$$\widehat{d} = d - \widetilde{C}_1 \int_0^{\pi} e^{\pi-\tau} f_1(\tau)\, d\tau + \widetilde{B}_2 \left[f_2(t) + N(t)\dot{f}_2(t)\right]_{t=0} + \widetilde{C}_2 \left[f_2(t) + N(t)\dot{f}_2(t)\right]_{t=\pi} =$$
$$= \begin{bmatrix} 0 \\ 1-e^{\pi} \\ -\pi e^{\pi} \end{bmatrix}.$$

Thus, since $\widehat{d} \in \operatorname{ran}(Q)$, the solvability condition is satisfied, and the boundary value problem admits a solution. The solution can then be computed explicitly using formula (17).

The matrix $Q \in \mathbb{R}^{3\times 1}$ has full column rank, so we may use its left inverse as a generalized inverse (see Section 2). This yields

$$Q^g = (Q^\top Q)^{-1} Q^\top = \left((1-e^{\pi})^2 + (\pi e^{\pi})^2\right)^{-1} \begin{bmatrix} 0 & 1-e^{\pi} & -\pi e^{\pi} \end{bmatrix}.$$

Substituting this expression into the solution formula (17), and simplifying, we obtain the unique solution of the boundary value problem

$$x(t) = \begin{bmatrix} \sin t \\ \cos t \\ t \end{bmatrix}.$$

This confirms the applicability of Theorem 6 and illustrates the effectiveness of the standard canonical form in analyzing the solvability of boundary value problems for time-varying DAEs.

References

[1] Ascher U.M., Petzold L.R. *Projected collocation for higher-order higher-index differential-algebraic equations*, Journal of Computational and Applied Mathematics, 43:1 (1992), 243–259. https://doi.org/10.1016/0377-0427(92)90269-4

[2] Bai Y. *A modified Lobatto collocation for linear boundary value problems of differential-algebraic equations*, Computing 49:2 (1992), 139–150. https://doi.org/10.1007/BF02238746

[3] Ben-Israel A., Greville T.N. *Generalized inverses: theory and applications*, Berlin: Springer, 2006.

[4] Berger T., Ilchmann A. *On the standard canonical form of tim-varying linear DAEs*, Quart. Appl. Math. 71:1 (2013), 69–87. https://doi.org/10.1090/S0033-569X-2012-01285-1

[5] Campbell S.L. *One canonical form for higher-index linear time-varying singular systems*, Circuits Syst. Signal Process. 2 (1983), 311–326. https://doi.org/10.1007/BF01599073

[6] Campbell S.L. *A general form for solvable linear time varying singular systems of differential equations*, SIAM Journal on Mathematical Analysis, 18:4 (1987), 1101–1115. https://doi.org/10.1137/0518081

[7] Clark K.D., Petzold L.R. *Numerical solution of boundary value problems in differential-algebraic systems*, SIAM Journal on Scientific and Statistical Computing, 10:5 (1989), 915–936. https://doi.org/10.1137/0910053

[8] Kunkel P., Mehrmann M. *Differential-algebraic equations. Analysis and numerical solution*, EMS Publishing House, Zürich, Switzerland, 2006.

[9] Lamour R., März R., Tischendorf C. *Differential-algebraic equations: a projector based analysis*, Springer Science & Business Media, 2013.

[10] Lamour R., März R., Weinmüller E. *Boundary-value problems for differential-algebraic equations: a survey*, in: Ilchmann A., Reis T. (eds) Surveys in differential-algebraic equations III. Differential-Algebraic Equations Forum (2015), 177–309. https://doi.org/10.1007/978-3-319-22428-2

[11] Riaza R. *Differential-algebraic systems: analytical aspects and circuit applications*, World Scientific Publishing, 2008.

[12] Stöver R. *Collocation methods for solving linear differential-algebraic boundary value problems*, Numer. Math. 88 (2001), 771–795. https://doi.org/10.1007/PL00005458

Утешова Р. Е. АЙНЫМАЛЫ КОЭФФИЦИЕНТТЕРІ БАР СЫЗЫҚТЫ ДИФФЕРЕНЦИАЛДЫ-АЛГЕБРАЛЫҚ ТЕҢДЕУЛЕР ҮШІН ШЕТТІК ЕСЕПТЕР: СТАНДАРТТЫ КАНОНДЫҚ ФОРМАНЫ ҚОЛДАНА ОТЫРЫП ШЕШІЛІМДІГІН ТАЛДАУ

Бұл мақалада айнымалы коэффициенттері бар сызықтық дифференциалды-алгебралық теңдеулер үшін екі нүктелі шеттік есептердің шешілімділігі қарастырылады. Ұсынылған әдіс стандартты канондық форманы қолдануға негізделген, бұл жүйені жәй дифференциалдық теңдеуге және таза алгебралық бөлікке ажыратуға мүмкіндік береді. Тиісті параметр енгізу арқылы бастапқы есеп сәйкес сызықтық алгебралық теңдеулер жүйесінің шешілімділігіне келтіріледі. Нәтижесінде шеттік есептің шешілімділігінің конструктивті критерийі алынды. Ұсынылған әдістің тиімділігі нақты мысал арқылы көрсетіледі.

Түйін сөздер: шеттік есеп, айнымалы коэффициенттері бар сызықты дифференциалды-алгебралық теңдеу, стандартты канондық форма, жалпыланған кері матрица.

Утешова Р. Е. КРАЕВЫЕ ЗАДАЧИ ДЛЯ ЛИНЕЙНЫХ ДИФФЕРЕНЦИАЛЬНО-АЛГЕБРАИЧЕСКИХ УРАВНЕНИЙ С ПЕРЕМЕННЫМИ КОЭФФИЦИЕНТАМИ: АНАЛИЗ РАЗРЕШИМОСТИ С ПРИМЕНЕНИЕМ СТАНДАРТНОЙ КАНОНИЧЕСКОЙ ФОРМЫ

В статье рассматривается разрешимость двухточечной краевой задачи для системы линейных дифференциально-алгебраических уравнений с переменными коэффициентами. Предлагаемый метод основан на использовании стандартной канонической формы, позволяющей расщепление системы на обыкновенное дифференциальное уравнение и чисто алгебраическую часть. Путём введения подходящего параметра исходная задача сводится к исследованию разрешимости соответствующей системы линейных алгебраических уравнений. В результате получен конструктивный критерий разрешимости краевой задачи. Эффективность метода иллюстрируется на подробном примере.

Ключевые слова: краевая задача, линейное дифференциально-алгебраическое уравнение с переменными коэффициентами, стандартная каноническая форма, обобщенная обратная матрица.