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# Numerical implementation of solving a boundary value problem with parameter for Fredholm integro-differential equation

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Abstract. The boundary value problem with parameter for Fredholm integro-differential equation with degenerate kernel is investigated in this paper. The aim of the paper is to establish the solvability conditions, to construct analytical and numerical solutions of the considered problem. The basis for achieving the goal is the ideas of Dzhumabayev parameterization method, classical numerical methods of solving Cauchy problems and numerical integration techniques. A problem with parameters is obtained by introducing an additional parameter and a new unknown function. A system of equations with respect to parameters is compiled according to the initial data of the considered equation and boundary conditions. The unknown function is found as a solution of the Cauchy problem for the ordinary differential equation. The equivalence of the original problem and the problem with parameters, the conditions of unique solvability are established and the formula for finding an analytical solution is obtained. Test examples of finding analytical and approximate solutions of the original problem are given.

Keywords. integro-differential equation, boundary value problem, parametrization method, parameter, solvability.

Integro-differential equations are widely employed across various domains, serving as mathematical models for diverse processes in physics, biology, chemistry, economics, and

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other fields. Their significant role in analyzing processes has been emphasized in the monographes [1]–[4] and in a review of early studies dedicated to initial and boundary value problems for integro-differential equations. Different aspects of qualitative theory and approximate methods applied to integro-differential equations and problems for such equatons, as well as their applications, are detailed in works [5]–[24].

We consider the linear boundary value problem with parameter for Fredholm integro-differential equation

$$\frac{dx}{dt} = p(t)x + \varphi(t)\int\limits_0^T \psi(s)x(s)ds + f(t), \quad t \in (0,T), \tag{1}$$

$$\alpha_i x(0) + m_i\mu + \beta_i x(T) = d_i, \quad \mu, d_i \in R, \quad i = 1,2. \tag{2}$$

where the functions $p(t)$, $\varphi(t)$, $\psi(t)$, $f(t)$ are continuous on [0, T].

A solution to Problem (1)–(2) is a pair $(x^*(t), \mu^*)$, where the function $x^*(t)$ continuous on $[0,T]$ and continuously differentiable on $(0,T)$ and satisfies the integro-differential equation (1) and boundary conditions (2) at $\mu = \mu^*$.

The problem (1), (2) is investigated by the parameterization method [25]. Introducing the additional parameter $\lambda = x(0)$, and performing a replacement of the function $u(t) = x(t) - \lambda$ on $t \in [0,T)$, we obtain the boundary value problem with parameters

$$\frac{du}{dt} = p(t)(u+\lambda) + \varphi(t)\int\limits_0^T \psi(s)(u(s)+\lambda)ds + f(t), \quad t \in [0,T), \tag{3}$$

$$u(0) = 0, \tag{4}$$

$$\alpha_i\lambda + m_i\mu + \beta_i\Big(\lambda + \lim_{t\to T-0} u(t)\Big) = d_i, \quad i = 1,2. \tag{5}$$

A solution to Problem (3)–(5) is a pair $(\Lambda^*, u^*(t))$, where $\Lambda^* = (\lambda^*, \mu^*)$, $u^*(t)$ is continuously differentiable on $[0,T)$, and satisfying the equation (3) and the conditions (4)–(5) at the $\lambda = \lambda^*$, $\mu = \mu^*$. A solution to the initial problem (3)–(4) can be write in the following form:

$$u(t) = e^{\int_0^t p(s)ds}\int\limits_0^t \Big[p(s)\lambda + \varphi(s)\int\limits_0^T \psi(\tau)(u(\tau)+\lambda)d\tau + f(s)\Big]e^{-\int_0^s p(s_1)ds_1}ds, \quad t \in [0,T). \tag{6}$$

We set $\xi = \int_0^T \psi(\tau)u(\tau)d\tau$ and rewrite (6) in the next form:

$$u(t) = e^{\int_0^t p(s)ds}\int\limits_0^t \Big[p(s)\lambda + \varphi(s)\xi + \varphi(s)\int\limits_0^T \psi(\tau)\lambda d\tau + f(s)\Big]e^{-\int_0^s p(s_1)ds_1}ds, \quad t \in [0,T). \tag{7}$$

Multiplying both sides of (7) by $\psi(t)$ and integrating on the interval $[0,T]$, we obtain the linear algebraic equations with respect to $\xi \in R$:

$$\xi = \int\limits_0^T \psi(\tau)\, e^{\int\limits_0^\tau p(s)ds} \int\limits_0^\tau \varphi(s)\, e^{-\int\limits_0^s p(s_1)ds_1} ds d\tau \cdot \xi +$$
$$+ \int\limits_0^T \psi(t)\, e^{\int\limits_0^t p(s)ds} \int\limits_0^t \Big[p(s)\lambda + \varphi(s)\int\limits_0^T \psi(\tau)d\tau\lambda + f(s)\Big] e^{-\int\limits_0^s p(s_1)ds_1} dsdt. \quad (8)$$

Let $a(v,t) = e^{\int\limits_0^t p(t)dt} \int\limits_0^t v(s)e^{-\int\limits_0^s p(s_1)ds_1} ds$. We rewrite (8) in the form

$$\Big(1 - \int\limits_0^T \psi(t)a(\varphi,t)dt\Big)\xi = \int\limits_0^T \psi(t)\Big[a(p,t)\lambda + a(\varphi,t)\int\limits_0^T \psi(\tau)d\tau\lambda + a(f,t)\Big]dt. \quad (9)$$

Then according to (9) the parameter $\xi$ can be determined by the equality

$$\xi = \frac{\int\limits_0^T \psi(t)\Big[\Big(a(p,t) + a(\varphi,t)\int\limits_0^T \psi(\tau)d\tau\Big)\cdot\lambda + a(f,t)\Big]dt}{1 - \int\limits_0^T \psi(t)a(\varphi,t)dt}. \quad (10)$$

Substituting the right-hand side of (10) instead of $\xi$ in (7), we obtain the representation of the function $u(t)$ via $\lambda$:

$$u(t) = g(t)\lambda + h(t), \quad t \in [0,T), \quad (11)$$

where

$$g(t) = a(p,t) + a(\varphi,t)\frac{1}{\theta}\int\limits_0^T \psi(s)a(p,s)ds + a(\varphi,t)\Big[1 + \frac{1}{\theta}\int\limits_0^T \psi(s)a(\varphi,s)ds\Big]\int\limits_0^T \psi(\tau)d\tau,$$

$$h(t) = a(f,t) + a(\varphi,t)\frac{1}{\theta}\int\limits_0^T \psi(s)a(f,s)ds,$$

herewith

$$\theta = 1 - \int\limits_0^T \psi(t)a(\varphi,t)dt.$$

Then from (11) we have

$$\lim_{t \to T-0} u(t) = g(T)\lambda + h(T). \tag{12}$$

Substituting the right-hand side of (12) into the boundary conditions (5), we obtain the following system of linear algebraic equations with respect to parameters $\lambda$ and $\mu$:

$$\begin{cases} [\alpha_1 + \beta_1(1 + g(T))]\lambda + m_1\mu = d_1 - \beta_1 h(T), \\ [\alpha_2 + \beta_2(1 + g(T))]\lambda + m_1\mu = d_2 - \beta_2 h(T). \end{cases} \tag{13}$$

By denoting the matrix corresponding to the left-hand side of the system (13) by $Q$ and this system can be written as

$$Q \cdot \Lambda = F, \quad \Lambda = (\lambda, \mu), \tag{14}$$

where

$$Q = \begin{pmatrix} \alpha_1 + \beta_1(1 + g(T)) & m_1 \\ \alpha_2 + \beta_2(1 + g(T)) & m_2 \end{pmatrix}, \quad F = \begin{pmatrix} d_1 - \beta_1 h(T) \\ d_2 - \beta_2 h(T) \end{pmatrix}.$$

**Lemma.** The following assertions hold:
(a) the vector $\Lambda^* = (\lambda^*, \mu^*)$, composed by the values of a solution $x^*(t)$ to the problem (1)–(2) at the point $\lambda^* = x^*(0)$, satisfies the system (14);
(b) if $\tilde{\Lambda} = (\tilde{\lambda}, \tilde{\mu})$ is a solution to the system (14) and the function $\tilde{u}(t)$ is a solution to the Cauchy problem for integro-differential equation (3)–(4) with $\lambda = \tilde{\lambda}$, $\mu = \tilde{\mu}$, then the function $\tilde{x}(t)$, defined by the equalities: $\tilde{x}(t) = \tilde{\lambda} + \tilde{u}(t)$, $t \in [0, T)$, $\tilde{x}(T) = \tilde{\lambda} + \lim_{t \to T-0} \tilde{u}(t)$, is a solution to the problem (1)–(2).

Let us introduce the notations

$$\overline{p} = \max_{t \in [0,T]} |p(t)|, \quad \overline{\varphi} = \max_{t \in [0,T]} |\varphi(t)|, \quad \overline{\psi} = \max_{t \in [0,T]} |\psi(t)|, \quad \|Q^{-1}\| = \gamma.$$

**Theorem.** Let $\theta \neq 0$ and the matrix $Q$ be invertible. Then the problem (1)–(2) has a unique solution $(x^*(t), \mu^*)$ for any $f(t)$, $d_1$ and $d_2$, and the following estimate holds:

$$\|x^*\| \leq N \max(|d_1|, |d_2|, \|f\|),$$

where

$$N = e^{\overline{p}T}\Big\{\overline{\varphi}\Big[\frac{1}{|\theta|}\overline{\psi}\Big(e^{\overline{p}T} - 1 + e^{\overline{p}T}\overline{\varphi}\overline{\psi}\Big) + \overline{\psi}\Big] + 1\Big\}\gamma(1 + \max(\beta_1, \beta_2))\times$$

$$\times \max\Big\{1, Te^{\overline{p}T}\Big[1 + e^{\overline{p}T}\overline{\varphi}\frac{1}{|\theta|}\overline{\psi}\Big]\Big\} + e^{\overline{p}T}T\Big[\overline{\varphi}\frac{1}{|\theta|}\overline{\psi}e^{\overline{p}T} + 1\Big].$$

The proof of the theorem is given similarly to the proof of Th.2.1 of [24].

Let $\lambda^*, \mu^*$ be a solution to (14). We construct the next function:

$$\mathcal{F}^*(t) = \varphi(t)\Big(\xi^* + \int_0^T \psi(\tau)d\tau \cdot \lambda^*\Big) + f(t), \tag{15}$$

and define the solution to the boundary value problem (1)–(2) by the equalities:

$$x^*(t) = \lambda^* e^{\int_0^t p(s)ds} + e^{\int_0^t p(s)ds} \int_0^t \mathcal{F}^*(s) e^{-\int_0^s p(s_1)ds_1} ds, \quad t \in [0, T), \tag{16}$$

$$x^*(T) = \lambda^* e^{\int_0^T p(t)dt} + e^{\int_0^T p(t)dt} \int_0^T \mathcal{F}^*(t) e^{-\int_0^t p(s)ds} dt. \tag{17}$$

Further we consider two test examples on finding solutions of boundary value problems for Fredholm integro-differential equations with a parameter.

**Examples.**

Consider the following linear boundary value problem:

$$\frac{dx}{dt} = p(t) + 2t\int_0^1 (12s^3 + 30)x(s)ds + f(t), \quad t \in (0, 1), \tag{18}$$

$$2x(0) + 3\mu - 6x(T) = 53, \tag{19}$$

$$4x(0) - 11\mu + 9x(T) = -154. \tag{20}$$

**Problem A.**

Let $p(t) = 1, \quad f(t) = -t^2 - 305t - 12.$

We introduce the additional parameter $\lambda = x(0)$ and $\xi = \int_0^1 (12s^3 + 30)u(s)ds$, where $u(s) = x(s) - \lambda$, on $s \in [0, 1)$.

Then for the function $u(t)$ we have the equality

$$u(t) = e^t \int_0^t \Big[\lambda + 2s\xi + 2s\int_0^1 (12\tau^3 + 30)d\tau\lambda - s^2 - 305s - 12\Big] e^{-s} ds, \quad t \in [0, 1), \tag{21}$$

Multiplying both sides of (21) by $(12t^3+30)$ and integrating on the interval $[0,1]$, we have the linear algebraic equations with respect to $\xi$:

$$\xi = \int_0^1 (12t^3+30)\, e^t \int_0^t 2s\, e^{-s} ds dt \cdot \xi + \\ + \int_0^1 (12t^3+30)\, e^t \int_0^t \Big[1 + 2s \int_0^1 (12\tau^3+30) d\tau \Big] e^{-s} ds dt \lambda - \\ - \int_0^1 (12t^3+30)\, e^t \int_0^t \Big[s^2 + 305s + 12\Big] e^{-s} ds dt. \quad (22)$$

From (22), we determine $\xi$:

$$\xi = \frac{5}{89-60e}\left[\left(402e - \frac{2727}{5}\right)\lambda + \frac{12414}{5} - 1914e\right]. \quad (23)$$

We substitute (23) into the right side of (21) and obtain the representation of the function $u(t)$ through $\lambda$:

$$u(t) = e^t \int_0^t \left[1 + 2s \int_0^1 (12\tau^3+30)d\tau + \frac{10s}{89-60e}\left(402e - \frac{2727}{5}\right)\right] e^{-s} ds \lambda + \\ + e^t \int_0^t \left[\frac{10s}{89-60e}\left(\frac{12414}{5} - 1914e\right) - s^2 - 305s - 12\right] e^{-s} ds, \quad t \in [0,1), \quad (24)$$

Then the boundary conditions using (24) at $t \to 1$ lead to the following system of linear algebraic equations with respect to parameters $\lambda$ and $\mu$:

$$\begin{pmatrix} \frac{2(1227e-2609)}{60e-89} & 3 \\ \frac{7204-3261e}{60e-89} & -11 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} \frac{20598e-41599}{60e-89} \\ \frac{69029-35367e}{60e-89} \end{pmatrix}. \quad (25)$$

From (25) we find the values of parameters $\lambda^* = 7$ and $\mu^* = 19$ and from (23) we find the value of $\xi^* = -75$. Then using found $\lambda^* = 7$ and $\xi^* = -75$, from (14) we construct the function

$$\mathcal{F}^*(t) = 2t\Big(-75 + 7\int_0^1 (12s^3+30)ds\Big) - t^2 - 305t - 12 = -t^2 + 7t + 12. \quad (26)$$

From (15) we define the solution

$$x^*(t) = 7e^t + e^t \int\limits_0^t (-s^2 + 7s + 12)e^{-s}ds, \quad t \in [0,1).$$

Therefore, the unique solution to Problem A is a pair $(x^*(t), \mu^*)$, where

$$\begin{cases} x^*(t) = t^2 - 5t + 7, \quad t \in [0,1), \\ x^*(1) = 3, \\ \mu^* = 19. \end{cases} \tag{27}$$

If the right-hand sides of (16), (17) contain "non-breaking" integrals, then the solution to the problem (1)–(2) is found numerically.

**Problem B.**

Let $p(t) = e^{-t^2}$, $f(t) = (5t - 7 - t^2)e^{-t^2} - 310t - 5$.

Here we use the numerical algorithm of Dzhumabaev parametrization method [24] for solving Problem B. Accuracy of solution depends on the accuracy of solving the Cauchy problems. We provide the results of the numerical implementation of algorithm by partitioning the interval $[0,1]$ with step $h = 0.05$.

Exact solution of Problem B is the (27).

The differences between the exact and approximate solutions to Problem B are provided in the following Tables:

| $k$ | $t_k$ | $\widetilde{x}(t)$ | $x^*(t)$ | $\lvert x^*(t) - \widetilde{x}(t)\rvert$ | $k$ | $t_k$ | $\widetilde{x}(t)$ | $x^*(t)$ | $\lvert x^*(t) - \widetilde{x}(t)\rvert$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 7.0000006666 | 7 | 0.0000006666 | 10 | 0.5 | 4.750000808 | 4.75 | 0.000000808 |
| 1 | 0.05 | 6.7525006924 | 6.7525 | 0.0000006924 | 11 | 0.55 | 4.5525008042 | 4.5525 | 0.0000008042 |
| 2 | 0.1 | 6.5100007161 | 6.51 | 0.0000007161 | 12 | 0.6 | 4.3600007965 | 4.36 | 0.0000007965 |
| 3 | 0.15 | 6.2725007376 | 6.2725 | 0.0000007376 | 13 | 0.65 | 4.1725007847 | 4.1725 | 0.0000007847 |
| 4 | 0.2 | 6.0400007566 | 6.04 | 0.0000007566 | 14 | 0.7 | 3.9900007686 | 3.99 | 0.0000007686 |
| 5 | 0.25 | 5.812500773 | 5.8125 | 0.000000773 | 15 | 0.75 | 3.8125007482 | 3.8125 | 0.0000007482 |
| 6 | 0.3 | 5.5900007865 | 5.59 | 0.0000007865 | 16 | 0.8 | 3.6400007235 | 3.64 | 0.0000007235 |
| 7 | 0.35 | 5.372500797 | 5.3725 | 0.000000797 | 17 | 0.85 | 3.4725006944 | 3.4725 | 0.0000006944 |
| 8 | 0.4 | 5.1600008042 | 5.16 | 0.0000008042 | 18 | 0.9 | 3.3100006609 | 3.31 | 0.0000006609 |
| 9 | 0.45 | 4.9525008079 | 4.9525 | 0.0000008079 | 19 | 0.95 | 3.1525006232 | 3.1525 | 0.0000006232 |
| 10 | 0.5 | 4.750000808 | 4.75 | 0.000000808 | 20 | 1 | 3.0000005811 | 3 | 0.0000005811 |

| $\widetilde{\mu}(t)$ | $\mu^*(t)$ | $\lvert \mu^*(t) - \widetilde{\mu}(t)\rvert$ |
|---|---|---|
| 19.0000007178 | 19 | 0.0000007178 |

**Remark.**

1) at $m_1 = m_2 = 0$ the problem (1)–(2) becomes overdetermined;

2) at $\dfrac{a_1}{a_2} = \dfrac{m_1}{m_2} = \dfrac{b_1}{b_2} = \dfrac{d_1}{d_2}$ the problem (1)–(2) becomes indeterminate.

**Conclusion.** In this paper, based on the ideas of Dzhumabayev parameterization method, a linear boundary value problem with a parameter for an integro-differential equation with a degenerate kernel is investigated. The conditions of unique solvability of the investigated problem are established. Two test examples for finding analytical and approximate solutions are given.

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Бакирова Э.А., Искакова Н.Б., Кадирбаева Ж.М. ФРЕДГОЛЬМ ИНТЕГРАЛДЫҚ-ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУІ ҮШІН ПАРАМЕТРІ БАР ШЕТТІК ЕСЕПТІ ШЕШУДІҢ САНДЫҚ ЖҮЗЕГЕ АСЫРЫЛУЫ

Жұмыста өзегі айныған Фредгольм интегралдық-дифференциалдық теңдеуі үшін параметрі бар шеттік есеп зерттелінеді. Жұмыстың мақсаты зерттелінетін есептің бірмәнді шешілімділігін, аналитикалық және сандық шешімдерін құру болып табылады. Мақсатқа жету негізі болып Джумабаевтың параметрлеу әдісі, Коши есептерін шешудің классикалық сандық әдістері және сандық интегралдаудың тәсілдері жатады. Қосымша параметрді және жаңа белгісіз функцияны енгізу арқылы параметрлері бар есеп алынады. Қарастырылып отырған теңдеудің, шеттік шарттардың бастапқы берілімдері бойынша параметрлерге қатысты теңдеулер жүйесі құрылады. Белгісіз функция жәй дифференциалдық теңдеу үшін Коши есебінің шешімі ретінде табылады. Бастапқы есеп пен параметрлері бар есептің эквиваленттілігі, бірмәнді шешілімділігінің шарттары тағайындалады және аналитикалық шешімді табудың формуласы алынады. Берілген есептің аналитикалық және жуық шешімдерін табудың тесттік мысалдары келтіріледі.

*Кілттік сөздер.* интегралдық-дифференциалдық теңдеу, шеттік есеп, параметрлеу әдісі, параметр, шешілімділік.

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ЧИСЛЕННАЯ РЕАЛИЗАЦИЯ РЕШЕНИЯ КРАЕВОЙ ЗАДАЧИ С ПАРАМЕТРОМ ДЛЯ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ ФРЕДГОЛЬМА

В работе исследуется краевая задача с параметром для интегро-дифференциального уравнения Фредгольма с вырожденным ядром. Целью работы является установление условий разрешимости, построение аналитического и численного решения исследуемой задачи. В основу достижения цели легли идеи метода параметризации Джумабаева, классические численные методы решения задач Коши и приемы численного интегрирования. Введением дополнительного параметра и новой неизвестной функций получается задача с параметрами. По исходным данным рассматриваемого уравнения, краевых условий составляется система уравнений относительно параметров. Неизвестная функция находится как решения задачи Коши для обыкновенного дифференциального уравнения. Устанавливается эквивалентность исходной задачи и задачи с параметрами, условия однозначной разрешимости и выводится формула нахождения аналитического решения. Приводятся тестовые примеры нахождения аналитического и приближенного решения исходной задачи.

*Ключевые слова.* интегро-дифференциальное уравнение, краевая задача, метод параметризации, параметр, разрешимость.